LIE Scales: Composing with Scales of Linear Intervallic Expansion

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For Simone and my boys

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The completion of this work would not have been possible without the contributions of the musicians who performed, recorded and/or engaged with my LIE scalic inspired pieces: Amelie Donovan (flautist), Daniel de Gruchy-Lambert (trumpeter), Julien Kottukapally (pianist), Katherine Ellis (vocalist), Matt Baker (harmonicist), Michael G. Ronstadt (cellist), Naomi Bayley (saxophonist and clarinettist), Richard Gibbons (bassist), and Steven Mai (trombonist).

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Abstract

This thesis includes a portfolio of scored compositions with written commentaries and a list of all completed pieces (2017–2024) composed using LIE scalic principle. All compositions use extended fixed pitch fields (FPFs) as source scales and are primarily scored for acoustic instruments. LIE scales (scales of *Linear Intervallic Expansion*) were initially derived from my discovery of a unique correspondence between consecutive counting numbers (+1, +2, +3...) and Messiaen's “mode 2” scale {0, 1, 3, 4, 6, 7, 9, 10}. In brief, this compound-chromatic theory of scales combines Non-Octave-Repeating Scales (beyond interval cycles) with Axiomatic scale theory. I explain my development of LIE scales, addresses some of the perceptual aspects of these FPFs, catalogue numerous scales and draw a compositional conclusion. My structural methodology is informed by the work of Webern, Bartók, Schillinger and Slonimsky, for example, but transcends 12-tone theory per se and suggests an alternative approach to harmonic dualism, whilst providing a rich generative vein for compositional development.

I explore abstract harmonic polarity by using extended *anti*/*complimentary* scales, treating melody and timbre as emergent entities rather than structural prerequisites, and research how harmonic meaning and our awareness of octave equivalence can be enhanced or avoided through composing with compound LIE scalic structures. This thesis should be of interest to any composers working with synthetic mathematically derived patterns, and musicologists specialising in early 20^{th} Century compositional approaches. LIE scales could also be used as a repository of alternative scalic ideas for improvisational purposes. Future research might explore LIE scalic principles *microtonally* and with regard to *granulation*, a *spectral centroid*, and *a-spatial* (or *medial*) theories of auditory perception.

List of portfolio scores

The online Portfolio of compositions (https://sonicimmersiontheory.com/portfolio/) contains direct links to PDF scores, demonstrative audio files, and some videos.

Music for Player Piano (Nos. 1, 3 and 6) (2017)
Duration: 9' 39"

74 pitches 4 hands 1 piano (2017)
Piano Duet
Duration: 4' 24"

A gilded cage (2017)
Scored for solo flute
Performed by Amelie Donovan
Duration: 5' 00"

Outographic No.1 (2018)
Scored for Cl., Bsn., Trp., Trb., Vln., Db., and Perc. (Glk. and Xyl.)
Trombone performed by Steven Mai, Clarinet by Naomi Bayley
Duration: 4' 20"

Notes of Protest (2019)
Duet for Cello and (virtual or live) B♭ Clarinet
Cello performed by Michael G. Ronstadt
Duration: 2' 12"

3313133 (parts I, II, and III) (2021)
Scored for Marimba, Vibraphone and Piano
Duration: 7' 15"

Structural Declaration No.1 (a piano concerto) (2021)
Scored for Large Ensemble and Piano
Duration: 9' 00"

Queuing for gristle and bone (2022)
Scored for B♭ Trumpet and Piano
Both parts performed by Daniel de Gruchy-Lambert
Duration: 4' 04"

An Anthem for Libertatia Part III (2022)
Scored for Large Ensemble (Small Orchestra) and Choir (SATB)
Soprano and Alto parts performed by Katherine Ellis,
Tenor and Bass parts performed by the composer
Duration: 7' 38"

Symphony No.2 (2022)
Scored for 1 Picc., 3 Fl., 2 Ob., Cl., B.Cl., Bsn., CBsn., 2 F.Hrn., Tpt., Tbn., B.Tbn., Tmp., Sn., Cym., Guit., Pno., Synth (pre-recorded), Vln., Vla., Vc., Cb
Duration: 21′ 15″

An Apology To The Ocean (Ymddiheuriad I'r Cefnfor) (2023)
Scored for Tenor Saxophone and Trombone
Performed by Naomi Bayley and Steven Mai
Duration: 6′ 16″

Biting the Bullet between Gritted Teeth (2024)
Written for Harmonica, Bass and Drum Kit
Performed by Matt Baker, Richard Gibbons, and the composer
Duration: 6′ 20″

String Quartet No. 1 (Where's Noah?) (2024)
Movement No. 1
Duration: 4′ 50″

Glossary of terms and abbreviations

Centrosymmetric
Registrally *fixed* symmetrical pitch fields have historically been explored under the auspices of symmetrical inversions, invertible counterpoint, mirror forms, and palindromes. The term *centrosymmetric* is preferred here as (i) musical literature often tends to trivialise or dismiss these forms; although very few composers and/or theorists have *not*, to varying extents, delved into this concept in one way or another, and (ii) in an ET environment, this term encompasses what (e.g.) Howard Hanson describes as “simple involution… [and] isometric sonorities” (1960, pp. 18–19). *

Equal Temperament (ET)
Our contemporary ET (piano) tuning system, in which the frequency ratio between adjacent tones is $\sqrt[12]{2}$.

Fixed Pitch Field (FPF)
This refers to any abstract source scale that is registrally specific/static, a *fixed* adaptation of Nauert's “pitch field” (2003, p. 181).

Interval Class (IC)
Generally, the ICs referred to here are *ordered* pitch-class intervals within an extended (unordered) intervallic frame. Minor Second = IC1, Perfect Fifth = IC7, Major Seventh = IC11 etc, (e.g.) G4 to B♭5 = 15. IC collections are written using integer notation within square brackets, i.e. [2, 1, 3] refers to a Maj2nd + min2nd + min3rd over a tritone [6] span.

Inverted Scale (I)
The inverse (descending ↓) variant of any given *fixed* prime scale, regardless of 12-tone partitioning.

LIE Scales
Scales of *Linear Intervallic Expansion*.
A term I began using in 2017 to describe any collection of intervals that may extend “beyond”, and do not necessarily repeat “at”, the octave. This is pronounced *Lie* (as in the direction, or position in which something lies); not to be confused with the mathematical Lie (pronounced *Lee*) group, other than that they are non-commutative (the order of entry directly affects the scalic outcome).

Mathemusical
A term first coined in the 1990's Journal of Mathematics and Music, used to balance the Leibnizian perspective of “music as exercitium arithmeticae” by considering mathematics as “exercitium musicae" (Andreatta and Agon, 2009, p. 63).

Modular (Mod)
This refers to any base number under consideration, where 1 = a semi-tone. E.g. a binary (two pitch) collection = Mod 2, and dodecaphonic music is simply Mod 12.

Octave Equivalence (OE)
An underlying premise of this thesis is that the harmonic dyads C3+C5 do not sound the same as (e.g.) C3+C4 or C4+C5, and B3+C5 sounds completely different to B3+C4. Hence the need for a registrally extended 12-TET OE+ syntax.

Outographic (OG)
The *anti*, or *complementary* collection of pitches that do not belong to any given *LIE* scale. The term *Outograph* is borrowed from Ted Joans (1928-2003), who used this term in relation to the process of cutting out the central subject of a photograph or image (see Tate Modern, 2022, p. 132). The relationship between an *OG* and its P+I *source* scale might be thought of as an auditory analogue of visual *figure-ground* perception.

P+I
This refers to any *fixed* (↑+↓) combination of prime and inverted scales.

Palindromic Prime (PP)
Whereas the traditional means of defining a Prime scale (Forte, 1973, pp. 179–181) (Rahn, 1980) is to (basically) analyse all the

internal intervallic permutations, then collate the revealed ICs (small to large), the PP defines the minimal span of any given P+I PC set/collection. This remains in accordance with Lewin's concept of directed intervallic distance (Lewin *et al*., 1977, p. 196), while approaching Forte's "Z-relation" (Mandereau *et al*., 2011). Obviously, any asymmetrical P scale combined with its inverse (I) will result in a centrosymmetric set, therefore P+I = an extended PP.

Pitch Class (PC)
In the light of David Lewin's "transformations" (1987), ICs are perhaps more pertinent to any/all harmonic enquiries. Both pitch names and corresponding integers are here written in curly brackets, where, from (e.g.) "C", {C, F, G} = {0, 5, 7}.

Prime Scale (P)
This initially refers to any *fixed* ascending (↑) scale regardless of 12-tone partitioning. Once a P+I LIE scale has been generated, the term *Prime* is then/also used to differentiate the LIE from its corresponding OG scale.

STET (semitones of equal temperament)
This term is here used to describe the semi-tonal span (or cumulative size) of any scale. I.e. C4 to G4 = 7-STET, C3 to D4 (a ninth) = 14-STET, A1 to E♭7 (and vice versa) = 66-STET. Primarily, this relates to *compound* 12-tone+ extensions. STET is therefore an ET "catch all" term for *span*, *range, gamut* and/or *ambitus*.

*The *black* accidentals are here consistently described as {C♯, E♭, F♯, G♯, B♭}.

Preface

Intervallic distance (both temporal and harmonic) is intrinsic to musical cognition. Polyrhythmic experimentation (the 3:2 hemiola and beyond) is clearly more meaningful when performed over a strict (i.e. countable or measured) time base, but to what extent might intervallic source scales derived from counting numbers (or integers) influence our perception of *harmonic* (vertical) intervals?

> Each interval used in music corresponds to a certain numerical proportion and, since... each harmonic connection can be composed by numerically described intervals, each composition can finally be understood and analytically recognized as an arrangement of uniquely determined relations of numbers (Martin Vogel cited in Wille and Wille-Henning, 2008, p.9).

The LIE scales explored in this thesis were initially (primarily, but not exclusively) derived from progressions of consecutive counting numbers. I began studying drum rudiments (which included workshops with James Blades) in the early 1970s, and have intermittently worked as a drummer ever since. Consequently, set theories and pitch/interval counting processes in general have long been of interest to me as a composer,[1] and in order to avoid getting dragged into discussing the *linguistic*-related emotive qualities of music, beyond the idea that "music sounds the way moods feel" (Pratt, 1931, p. 203), predominantly instrumental compositions are here presented. My compositional methodology is, in a primitive naive sense, analogous to that of Charles Ives, who, as a boy, played drums in his father's brass band and would practice his drum parts on a piano. In Ives's own words,

[1] I previously experimented with twelve-tone techniques, FPFs and mutating/rotating tetrachords, hexachords and tropes (Josef Matthias Hauer's concept of tropes rather than strict or integral serialisation per se). Hauer's 44 rotational tropes can be found in Dixie Lynn Harvey's translation of Hauer's *Vom Melos Zur Pauke: Eine Einfuhrung In Die Zwolftonmusik* (Harvey, 1980, p. 141).

> I got to trying out sets of notes to go with or take off the drums… They had little to do with the harmony of the piece, and were used only as sound-combinations… often I kept a different set of notes going in each hand… for accents the hands would go usually in opposite directions, the right hand up, the left hand down… when practicing… it didn't seem to bother the other players… if… I would keep away from triads etc. that suggested a key… I just mention the above… to show how the human ear (not one but all) will learn to digest and handle sounds, the more they are heard and then understood (Ives, 1973, pp. 42–43).

This is primarily a creative artistic undertaking, and I hope that listening to the new music generated from LIE scales (both the utilitarian MIDI realisations and, where possible, recorded performances)[2] provides greater insight into the concepts discussed (and vice versa). All the LIE scales collated throughout this thesis (and especially those listed in Appendix II) might also be thought of as FPF source scales, upon which to rhythmically improvise.[3]

"The musician feels mathematics, the mathematician thinks music"
(James Joseph Sylvester, cited in Lynch, 2015)

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[2] Throughout this research (2017-2024) I approached hundreds of musicians, and many of the thirty pieces listed in Appendix III were composed for specific "calls for scores". However, due to various practical and funding limitations, not all of the compositions in the portfolio resulted in a performed recording.

[3] Beyond the scope of this thesis is a (broadly) substantivalist view that the countable aspects of auditory perception might relate to a "physical substrate of consciousness (PSC)" (Tononi *et al*., 2016, p. 450): I refer to this as a Sonic Immersion Theory (SIT). *Sonic* as in a hierarchical or stratified relationship to sound or sound waves, i.e. Feneyrou describes "Scelsi's primitivism [as] the hedonism of unprecedented combinations of sonic strata" (2001), and *Immersion,* regarding the 24/7 mental involvement that audition necessitates. Giulio Tononi's *Integrated Information Theory (IIT)* of consciousness (Tononi *et al*., 2016) suggests that neuronal activity in the brain is "identical to the quality of the experience" (Tononi *et al*., 2016, p. 451), and that "every experience exists intrinsically and is structured, specific, unitary and definite" (ibid., p. 450). If, what Tononi calls, the "physical substrate of consciousness (PSC)" (ibid.) is thought of in relation to the subliminally countable/rhythmic "monads" of Leibniz's "Pre-established Harmony and Causality" (McDonough, 2017), then (if nothing else) this rhythmicity suggests that "there are other virtual events enveloping actual movement, sensory perception, and chronological time… [and that] rhythmic time lurks in the unknown and 'unthought' dimensions of experience" (Ikoniadou, 2020).

Callender et al. suggest that musicians "commonly abstract away from five types of information: the octave in which notes appear, their order, their specific pitch level, whether a sequence appears right-side up or upside down (inverted), and the number of times a note appears" (2008, p. 346). LIE scales incorporate all five of these musical considerations, to varying extents, from a compound chromatic (rather than diatonic or enharmonic) basis.

Part 1: The countable foundations of LIE scalic construction

Imagine a piano keyboard of unlimited length. By simply counting piano keyboard pitches/semitones in ascending (↑) order, I found that the consecutive counting number (intervallic) sequence 0 + 1 + 2 + 3... (*ad infinitum*) will only produce the octatonic pitches (and intervals) of Messiaen's second mode (mode 2) of limited transposition {0, 1, 3, 4, 6, 7, 9, 10} [1, 2, 1, 2, 1, 2, 1, 2]. As I could not find any specific reference to this particular integer/interval correspondence in the musical literature, what began in 2017 as little more than a "Musofun" (Schillinger, 1946, p. 1640) counting exercise became the skeletal foundations of this enquiry.

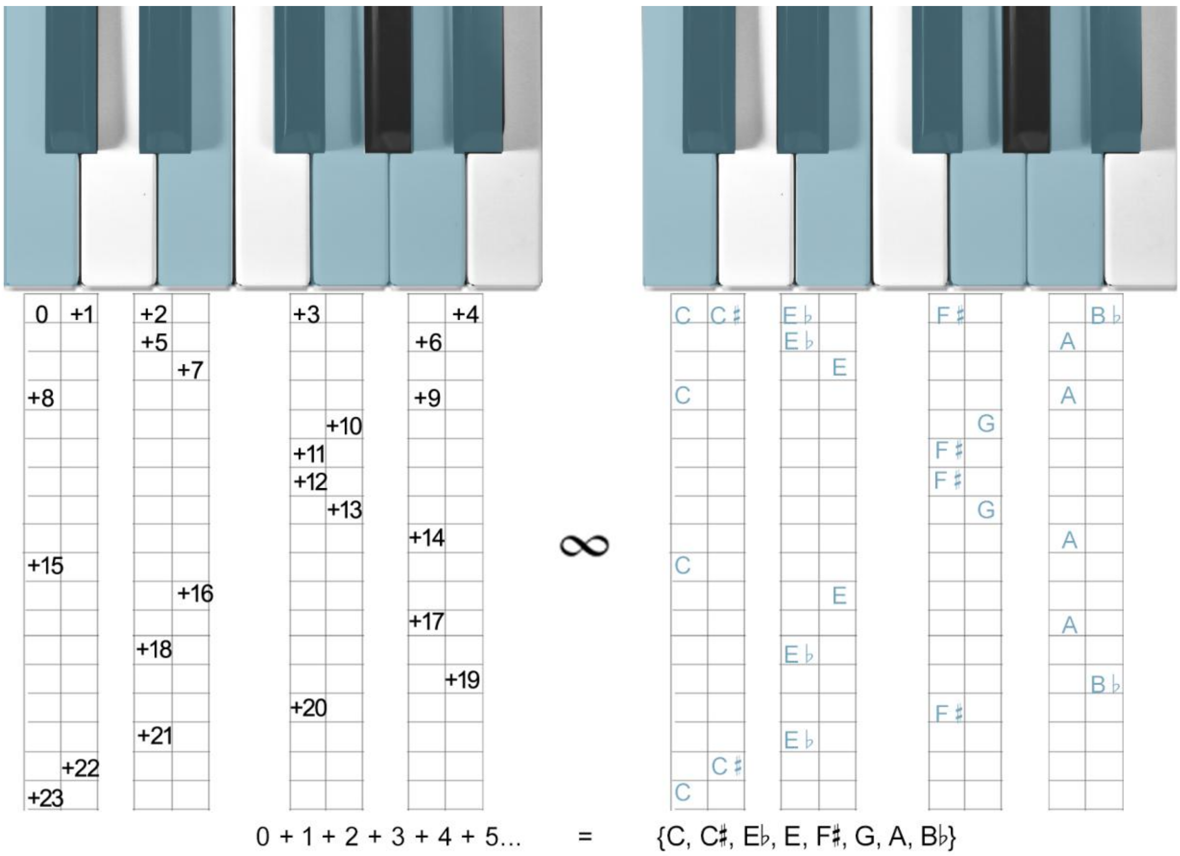


Figure 1: Consecutive counting numbers/Mode 2 Congruence

If, for example, C = 0, then 0 + 1 + 2 + 3 + 4 + 5 + 6 + 7... produces the pitches {C (+1) C♯ (+2) E♭ (+3) F♯ (+4) B♭ (+5) E♭ (+6) A (+7) E...}, an expanded rather than uniform {C, C♯, D, E♭....} semitonal scale. Intervallically, the extended sequence of triangular numbers (0, 1, 3, 6, 10, 15...) is produced by cumulatively collating consecutive counting numbers (0 + **1** = 1, 1 + **2** = 3, 3 + **3** = 6, 6 + **4** = 10...).[4]

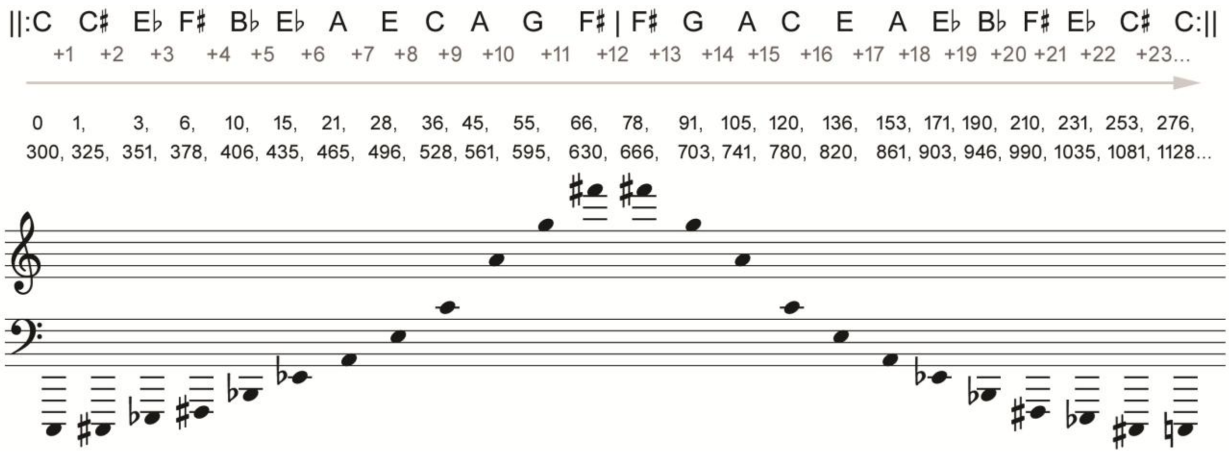


Figure 2: Scalic expansion by triangular numbers

Although the extended mode 2 (triangular number) scale in Figure 2 (above) is unidirectional and collated in ascending order only, the sequential order of the PCs inverts (changes direction) every sixth octave (72-STET), at the mid-point between the *all-interval* 66-STET/span (where, using the formula ***n* (*n*+1)/2**, *n* = 11) and the 78-STET (where *n* = 12). Consequently, this particular extended and invertible attribute of the chromatic twelve-tone series is not specifically mentioned by e.g. Slonimsky (1947), Bauer-Mengelberg and Ferentz (1965), Morris and Starr (1974), or Carter (2002). As this inversion occurs at an exponential

[4] Triangular numbers are, of course, not unique musicological terrain (Kak, 2004) (Burt, 2002); mathematicians would regard this as naive modular arithmetic, and only drummers might have bothered to carry on counting. Perhaps the process of *structurally* composing using integer/interval correspondences and the act/art of drumming (which I have been engaged with for 50 years) are both (variously) creative and cathartic means of dealing with *arithmomania* (obsessive counting).

(6th, 24th, 54th...) rather than strictly octaviated rate/distance, it is arguably less bound to OE partitioning. In OE PC rather than *pitched* LIE terms, the chart below (Figure 3) reveals the preponderance of minor third (diminished seventh) ladders (Van der Merwe, 1989, p. 120) within the above scale (Figure 2), which suggests a tritonal related (C-F♯ and E♭-A) "double axis of symmetry" (Roig-Francolí, 2021, p. 52), yet the tritone [6] is the least prevalent IC.

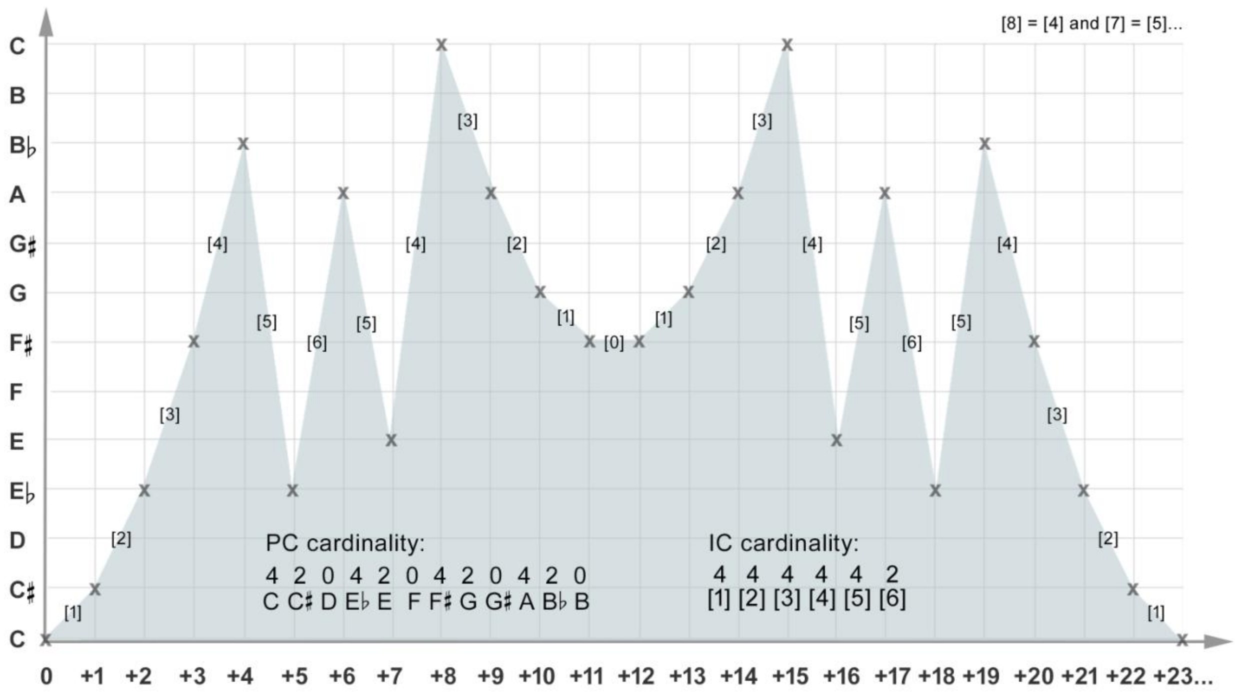


Figure 3: Plot Chart of the extended Mode 2 / counting number correspondence

To better understand any non-octaviated pitched/harmonic potentiality *beyond* the 12-STET species, I first listened to, looked at and counted modular sub-divisions *within* the dodecaphonic scale. I was intrigued to see what further auditory symmetries might emerge. In mod (or base) 12, Messiaen's octatonic mode 2 scale is essentially the cyclical repetition (or partition) of four successive mod 3 scales; (C, C♯, D) → (E♭, E, F) → (F♯, G, G♯) → (A, B♭, B), where only the first two pitches of each set are sounded. Using the counting number sequence 0 + 1 + 2..., it transpires that the number of utilised PCs in all modular sets from 1 to 12 (i.e. 1, 2, 2, 4, 3, 4, 4, 8, 4, 6, 6, 8) corresponds exactly to the sequence of "triangular numbers mod *n*" (OEIS: A117484) [5] in *The On-line Encyclopedia of Integer Sequences* (OEIS, 2022): see Figure 4 (below).

Mod (base)	Utilised PC's	No. of utilised PC's	Unused (OG) PC's
1	C	1	
2	C C♯	2	
3	C C♯	2	D
4	C C♯ D E♭	4	
5	C C♯ E♭	3	D E
6	C C♯ E♭ E	4	D F
7	C C♯ E♭ F♯	4	D E F
8	C C♯ D E♭ E F F♯ G	8	
9	C C♯ E♭ F♯	4	D E F G G♯
10	C C♯ E♭ F F♯ G♯	6	D E G A
11	C C♯ E♭ E F♯ B♭	6	D F G G♯ B
12	C C♯ E♭ E F♯ G A B♭	8	D F G♯ B

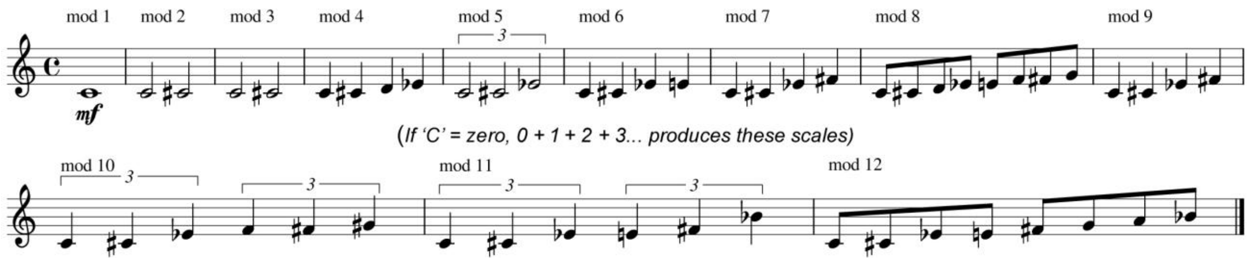


Figure 4: Utilised PCs for modular sets 1 to 12
(MP3 here; durational equivalence is applied to each modular set)

[5] Throughout this thesis, OEIS sequences are referred to by their catalogue number, i.e. A117484 in this instance.

IC's	Utilised (ascending) PC's	No. of utilised PC's	Octaves of resolution
0+1...	C C♯ D E♭ E F F♯ G G♯ A B♭ B	**12**	1
0+1+1...	C C♯ D E♭ E F F♯ G G♯ A B♭ B	**12**	1
0+1+2...	C C♯ E♭ E F♯ G A B♭	**8**	1
0+1+3...	C C♯ E F G♯ A	**6**	1
0+1+4...	C C♯ F F♯ B♭ B E♭ E G♯ A C♯ D F♯ G B C E F A B♭ D E♭ G G♯	**12**	5
0+1+5...	C C♯ F♯ G	**4**	1
0+1+6...	C C♯ E♭ F♯ D E♭ A B♭ E F B C F♯ G C D G♯ A E♭ E B♭ B F F♯	**12**	7
0+1+7...	C C♯ G♯ A E F	**6**	2
0+1+8...	C C♯ A B♭ F♯ G E♭ E	**8**	3
0+1+9...	C C♯ E♭ B♭ B G♯ A F♯ G E F D E♭	**12**	5
0+1+10...	C C♯ B C B♭ B A B♭ G♯ A G G♯ F♯ G F F♯ E F E♭ E D E♭ C♯ D	**12**	11
0+1+11...	C C♯	**2**	1
0+1+12...	C C♯ D E♭ E F F♯ G G♯ A B♭ B	**12**	13
0+1+13...	C C♯ D E♭ E F F♯ G G♯ A B♭ B	**12**	7

Figure 5: ICs (0+1) +1 to (0+1) +13
(MP3 here; durational equivalence is applied to each modular set)

Secondly, by expanding the counting number formula from +1+1… to +1+2…, +1+3…, i.e. semitone + semitone, semitone + tone, semitone + minor third, etc. (see Figure 5, above), the octave resolution sequence (1, 1, 1, 1, 5, 1, 7...) is directly correspondent to the "Numerator of n/12" (OEIS: A051724), [6] and the number of utilised PCs occurring in each of the nine (unidirectional) sets from 0+1+1 to 0+1+9 creates a recurring and centrosymmetric pattern: 12, 8, 6, 12, **4**, 12, 6, 8, 12.

Beyond the mathematical fact that "absolutely all rational numbers have digital sequences that eventually repeat" (Wolfram, 2002, p. 138), the emergence of the above mentioned centrosymmetric properties, at various levels, suggested that the *rate* (integral/intervallic distance), span or STET at which scalic sequences resolve (loop/repeat) and the resulting discrete/truncated scalic boundaries/vertices, [7] were worthy of further exploration. This is of interest because (i) the term *expanded tonality* has generally been applied to counterpoint as well as to various early twentieth-century hierarchical systems "whose principal feature is not, however, expansion" (Sachs and Dahlhaus, 2001), and (ii) *pantonality* might equally refer to a combination of systems as well as to a moveable tonic. Giovanni Albini points out that mathematics can be "useful... as a means for unique aesthetic aims and as an actual peculiar instrument that can potentially trigger new musical ideas" (2019, p. 54).

[6] This sequence is mentioned by A. A. Goldstein in "Optimal Temperament" (1977, p. 554). Various interesting integer/interval correspondences are listed in Appendix I of this thesis.
[7] In as much as all boundaries within (e.g.) Franz Brentano's concept of a continuum "are simply metaphysical parts of the whole of which they are the boundaries" (cited in Fréchette, 2017, p. 3).

LIE scalic antecedents; harmonic dualism and negative harmony

Thus far, LIE scales have here been treated as extended (12-STET+) *unidirectional* scales that may potentially be derived from any integer/intervallic sequence.[8] My next generative step was to explore the bidirectional combination of P+I scales (informed by the Second Viennese School; Schoenberg, Webern, Berg *et al*.) over *extended* (chromatic and compound but not necessarily octaviated) registral STETs, which inevitably leads to connotations of *negative harmony* such as Harry Partch's "Utonality" (1974, p. 69), Hugo Riemann's dualistic (over and under) "Klangs" (1896, p. 25), and Moritz Hauptmann's "double determination" (1888, p. 225).[9]

In recent years the term "negative harmony" was popularised by Jacob Collier (Lee, 2017), but the concept has historically been discussed in terms of harmonic "dualism". Steve Coleman suggests that he was the first to use the term "negative harmony" and, in the same Facebook post from 2018, Coleman provides a long list of theorists and musicians who have broadly explored this area; from Marchetto da Padova to Easley Blackwood (Coleman, 2018). I first discovered Coleman's "Symmetrical Movement Concept" (2015) a few months after I began developing LIE scales (and Collier much later), and although there are obvious similarities, there are also differences (not least STET limit, and OG usage) to our structural approaches. However, discovering Coleman's work in this area (along with that of Hauptmann, Sigfrid Karg-Elert, Bartók, and Webern; specifically but not exclusively) was

[8] Natural numbers are simply a bijection on a line that can have a one-to-one (or alternative) correspondence with auditory elements, and although cyclical forms, keys, fifths, and fourths etc, allude to a metaphysical idea of infinity; they are ostensibly a means of discussing linear sub sets of partitioned and repeated sounds. Conversely, what Messiaen refers to as modes of *limited* transposition might be thought of as cyclical forms with a multiplicity of potential zeros or start/end and middle/axis (beyond the tritone and mod 12) points.

[9] See Westerby for an overview of pre-twentieth century "dual theory" (1902, pp. 22–28) and a criticism of Hauptmann's use of Hegelian dialectics (the *reconciliation of opposites*) from a tonal perspective (1902, pp.29–70).

reassuring, in that I was not "hermetically pursuing something that nobody else is interested in" (Croft, 2015, p. 10), but with regard to originality, it hopefully encouraged me to employ more "idiosyncratic *musical* solutions to problems of *musical* material that arise only during the process of composition" (ibid.).

> It should certainly not be overlooked that to a certain extent a succession of notes is already a musical idea and that the number of such ideas is greater, the more notes there are available (Schoenberg, 1922, p. 25).

Numerous musical and extra-musical influences have informed the development of my extended P+I chromaticism. Antecedents of note are Schillinger's expansion of pitch-scales (1946, p. 133) and Nicolas Slonimsky's *Quadritonal* to *Sesquiquinquetonal* progressions of two to eleven octaves (1947, pp. 91–136); although both are ostensibly reliant on octaviated pitch space. Howard Hanson's "Projection by involution" (1960, p. 225), Vincent D'Indy's "superior" and "inferior" resonances (cited in Montgomery, 1946, p. 155), and Bernhard Ziehn's "inversion of Intervals" (1907, p. 2) have also helped to formulate this concept. Aside from (arguably) Bartók, Boulez,[10] and Xenakis's "Sieves" (1990),[11] few contemporary composers seem to have specifically explored centrosymmetric *non-octaviated* >12-STET ET FPFs,[12] i.e. from a compound and centrosymmetric rather than microtonal, spectral, or discrete Fourier transform (DFT) perspective; and none have addressed this in relation to an amelodic quasi-Leibnizian temporal sensibility (see Bizzell-Browning, 2024). E.g. Huovinen's

[10] Although Boulez's pitch-class multiplication techniques (Heinemann, 1998) (Losada, 2014) create pitch domains that registrally extend beyond the octave, no inverse centrosymmetric component is *intrinsically* involved.

[11] "Sieve Theory" is earlier mentioned by Xenakis in "Towards a Metamusic" (1970, p. 14)

[12] If Luciano Berio's 13 note tone row springs to mind (as used in *Nones*, 1954), it is worth pointing out that, although symmetrical [343315|513343] {B, D, B♭, G, E, E♭, A♭, D♭, C, A, F♯, D, F}, see Neidhöfer (2009, p. 306), Berio's conjunct heptachords (with the duplicated "D") are not (as is the case with LIE scales) registral extensions of the 12Tet species.

“Pitch-Class constellations” (2002) and Pärt’s “tintinnabuli technique” (Tokun, 2011) are reliant on melodic considerations, Lerdahl’s pitched “step distance” (1988, p. 322) is reliant on tonality, Schat’s “Tone Clock” (1993) is (although symmetric) an extension of the OE dodecaphonic clock face, and Brown’s “Hyperscales” [13] (2019) do not employ any intrinsic centralised (P+I) axial reciprocity. Unlike (e.g.) Hermann Schröder (1902) and Julian Anderson (2019), my arithmetic approach largely relates to Ziehn's (and Partch's) rejection of “the need for acoustical justification for a harmonic theory” (Nolan, 1983, p. 63).

> ...sensuous pleasures are really confusedly known intellectual pleasures. Music charms us, although its beauty only consists in the harmonies of numbers and in the reckoning of the beats or vibrations of sounding bodies, which meet at certain intervals, reckonings of which we are not conscious and which the soul nevertheless does make (Gottfried Wilhelm Leibniz, 1714, cited in Leibniz, 1908, p. 216)

Although it is generally thought that Leibniz’s “unconscious” counting (or reckoning) primarily refers to harmonic beat frequencies, Leibniz’s relentless pursuit of increasingly determinate finite numerical sequences often involved relatively simplistic additive counting procedures that began with the Mersenne sequence in 1666 (see Appendix I of this thesis). Leibniz’s use of the Mersenne sequence is perhaps indicative of a more literal interpretation of *countability* in relation to chromatic musical scales. Through brute-force counting I found that, in OE terms, all centrosymmetric (palindromic) scales contain an equal amount of common major and minor triads. [14]

[13] Clough *et al*. use the term “hyperscales” for PC sets that satisfy one of two sets of axioms derived from diatonic sets and Indian *gramas* (1997, p. 83). Their *hyperdiatonic* sets were found for PC sets of (what they call "U" for universe) 20-STET spans (1997, p. 91), and *hypergramas* (1997, p. 94) were found for 30-STET PC spans.

[14] To cross-reference this, see the *Common Triads* section for each of Ian Ring's 52 palindromic scales (Ring, 2016). Ring's exceptional online 12-tone resource is referred to throughout this thesis.

LIEs in relation to Sieves

As with Xenakis's *Sieves*, the compositional outcomes derived from abstract pitched or "outside-time" (Xenakis, 1970, p. 4) LIE scalic material is more important than the generative theory, but the purpose of both is to "materialize movements of thought through sounds, then to test them in compositions" (Xenakis, 1992, p. ix). Whereas both LIE scales and Sieves are generated from a zero pitched point of origin along a number line, [15] both allow for the application of nested transformations, what Xenakis refers to as "metabolae" (1970, p. 6), and both are generated from additive integers with a possible inverse (I) variant; only LIE scales are *essentially* cumulative and *intrinsically* P+I combinatorial. In other words, LIE scales are centrosymmetric with regard to any complete span/ambitus, whereas the individual internal symmetries of Sieves are derived from residual "spatial identities" (Xenakis, 1992, p. 268). LIE scales are a much simpler means of pitch generation, and therefore perhaps a more readily perceived means of exploring OE+ extensions in general. Both Sieves and LIE scales are however dependent on the premise that "the perception of a basic intervallic unit... is fundamental to all musical perception" (Jones, 2001, p. 254), but to what extent do these units (pitch chroma or protos chronos) [16] need to add up to, or necessarily repeat at, the octave (pitch height)? LIE scales might be best thought of as one dimensional Cartesian coordinates on a Y axis number line that extends (↑↓, '+' and '-')

[15] Musical notations (of all types) are effectively mathematical entities/instructions on a number line that define the frequencies and durations of a musical concept, and it is worth noting that within Category Theory, "there is no such thing... as 'the' natural numbers. However, it can be argued that there is such a thing as 'the concept' of natural numbers" (Marquis, 2020). See also "The Meaning of Category Theory for 21st Century Philosophy" (Peruzzi, 2007).

[16] Pitch chroma broadly relates to the perception of distance between pitches and "protos chronos" (Levin, 2007, p. 410) or "chronos protos" (Williams, 1911, p. 28) refers to Aristoxenian "primary time", the smallest useful division of sound, or distance between pulses. Williams suggests that the "whole of the Aristoxenian theory of rhythmopoeia is founded on the various uses of the chronos protos, a term which can conveniently be translated as 'primary time'" (1911, p. 28), and that with "very rapid music the conductor will embrace several primary times in a beat" (ibid.)

from an axial centre (zero). Perhaps my concept of numerically (integer/interval) extended reciprocity is more related to Ernst Levy's Polarity theory or absolute "telluric gravity" (1985, p. 15), where "the fulcrum, the point of balance between the opposites, the unmanifested still point" (Godwin, 1982, p. 377) is here explored through P+I extension.

P+I combinations

All discrete (i.e. truncated) and registrally extended P+I scalic collections result in a central axial (zero) point of reference within a frame; a traditionally 12-tone or octave sized container. Regardless of the specific intervals/integers involved, there are only two means of creating combinatorial (↑↓) P+I centrosymmetric pitch collections:

- No. 1 - From the centre (wherever that might be) outwards, and
- No. 2 - The superimposition of *Prime* (P) ascending (bottom to top) + *Inverse* (I) descending (top to bottom) scales, resulting in the former (No. 1).

Through registral extension, the generative principles of LIE scales differ from those of twelve-tone row/matrix construction in various ways. Having generally chosen not to take pitch entry order into consideration (when composing) and to use fixed distance STETs as a registrally expanded (>12-STET) limit, transposition becomes ostensibly irrelevant as an initial generative device. Similarly, with P+I constructs, the last pitch is of as much significance as the first, therefore the traditional 12-tone means of defining (or numbering) an inverted scale by its chromatic distance from zero is also irrelevant. For explanatory purposes, if we simply number any prime row from 1–12 (from first to last pitch), of the 4 possible (non-transpositional) permutations for each tone-row derived from the prime (P), retrograde (R), inversion (I), and retrograde inversion (RI) scales (see Figure 6), over any specific STET (in this example the combined cumulative span is 54 semitones), only P1 and I12 are relevant as P1↑ = R↓ and I12↓ = RI↑. This can be shortened to P+I = a combinatorial bidirectional (↑↓) LIE scale.

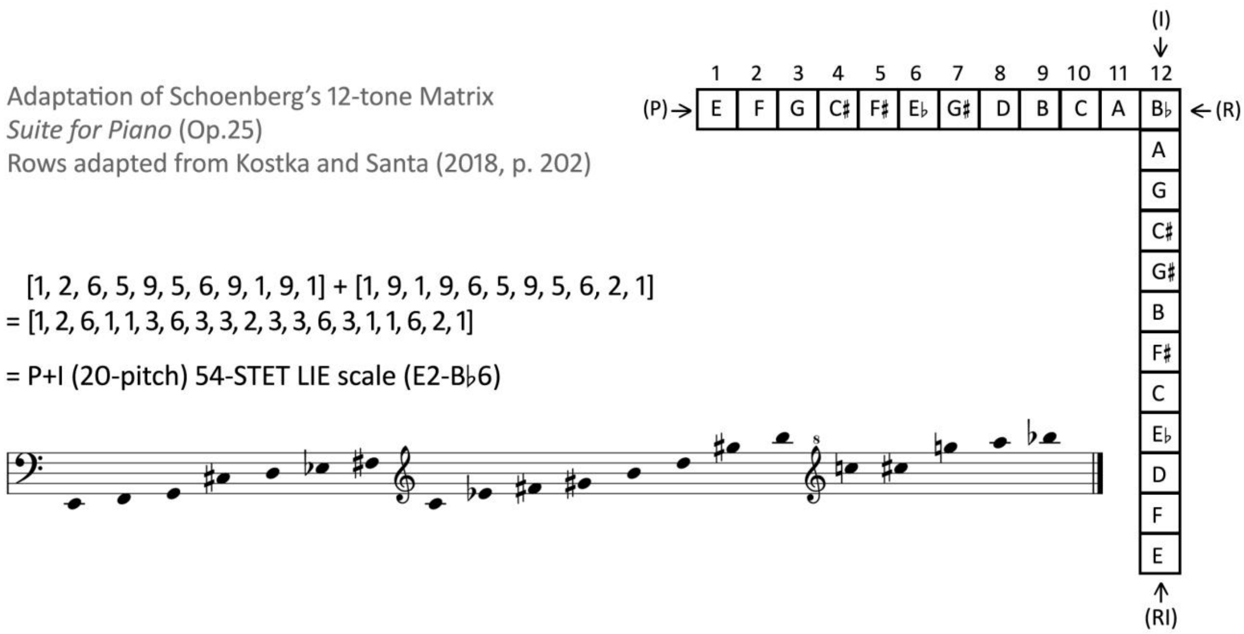


Figure 6: LIE scalic adaption of Schoenberg's P+I rows (Op.25)

The ET chromatic scale is of course a "fixed" concept, and the extent to which scalic expansion might inform dualist concepts of negativity with balance is debatable. However, the concept (and abstract harmonic potential) of a registrally extended P+I scale is intriguing. Is a 19-STET (twelfth) expanded ET scalic construct (e.g. from C4 to G5, or G3 to D5 etc – an octave + a fifth) as relevant (numerically, at least) as an equal 19-edo (63 cent) division of the octave? Rather than a series of stacked octaves pertaining to a tonal tonic/dominant variant, a tone row, or Stravinsky's "transpositionally related" polarity (Kielian-Gilbert, 1982,

p. 210), the idea of countable/numerical harmonic distance(s) is here both fundamental, and a limiter, one that potentially places (or replaces) a sense of tonic at the axial *centre* of a generative abstract structure; as arguably was often the case with *tonalité antique.*[17]

Refining the >12-STET P+I LIE scalic concept

LIE scales can be broadly described as compound (larger than an octave) "networks" with "axial isography" (Stoecker, 2002) derived from truncated vertices. However, although *gravitational* balance, isographic "contrary motion" (Segall, 2010) and inversional PC balance around "a *pair* of antipodal centres" (Lewin, 1968, p. 1) are clearly valid considerations for symmetrical scales of any span, as with "contextually defined octaves" (Rogers, 1968) that primarily refer to microtonal subdivisions, and "contextual-inversion spaces" (Straus, 2011) that relate to chains and pitch commonality, none of the above *specifically* accommodate the theoretical potential of a >12-tone FPF. Quinn describes chords as "combinatorial... sets of pitch classes drawn from an underlying universe" (2006, p. 118), and I tend to think of LIE scales as combinatorial extended chords that map out unique terrains. Rather than debating which particular PC set or group-theoretic attributes might be applicable, and the resulting (often semantic) differences between a synchronically presented chord and scale, I will briefly address some of the characteristics of P+I LIE scalic constructs.

[17] Of interest in this respect are Kubik's African "Span process… the projection of the idea of equidistance upon auditory materials" (2005, p. 184), and Dumbrill's suggestion that the ascending order of Western pitches is derived from "an older scheme where the tonic note was the axis of symmetry" (2019, p. 3), which perhaps explains why the ancient Greeks required a double octave systēma, and may also relate to Palisca's point that the Seikilos Epitaph (the only surviving intact ancient Greek *score*) deploys the mese in terms of "position" rather than "function" (2006, p. 77).

(i) The Yavorskyan axis

In its simplest form, Boleslav Yavorsky's "double symmetrical system" (Lupishko, 2016, p. 80) [18] can be said to describe the minimal semitonal intervallic span/value of a centralised *axial* range.

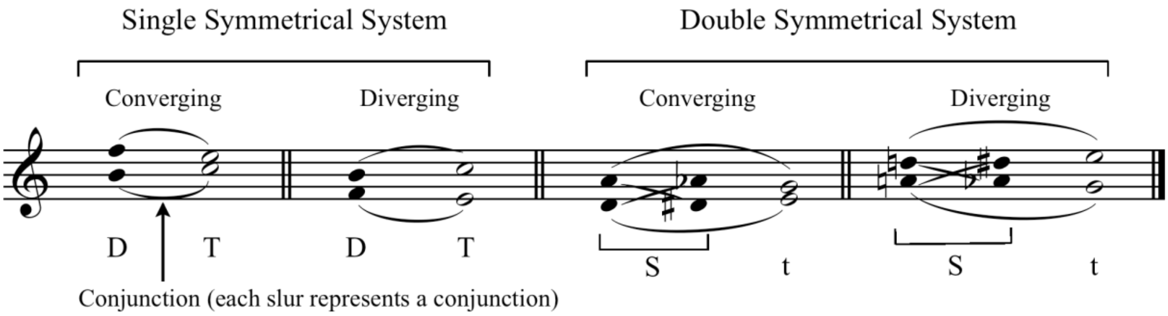


Figure 7: Yavorsky, Single and Double Symmetrical Systems (Ewell, 2012).

In the example above (Figure 7), the two measures of the single symmetrical system (on the left) both extend symmetrically from a *single* central pitch; bar 1 = {B C (D) E F} [1, 4, 1], and bar 2 = {E F (G♯) B C} [1, 6, 1]. With the double system (on the right), the central identity is expanded from one to two pitches, simply *doubled*; bar 3 = {D D♯ E (F/F♯) G A♭ A} [1, 1, 3, 1, 1] and bar 4 = {G A♭ A (B/C) D D♯ E} [1, 1, 5, 1, 1]. In other words, all centrosymmetric scales with an *even* span/STET (e.g. a major third and an octave) have a single axial pitch identity, and all *odd* span scales (e.g. a minor third, fourth and fifth) have a double axial identity. In this thesis, all single axial pitch points will be referred to as Y1 (Y for Yavorsky) and those with a double pitch as Y2. For example, all 14-STET (ninth) scales from (e.g.) C4 to D5 will have a central Y1 at G4 {C4 [7] G4 [7] D5}, and all 17-STET (eleventh) scales from (e.g.) C4 to F5 will have a central Y2 at G♯4/A4 {C4 [8] G♯4/A4 [8] F5}. This axial identity is a constant,

[18] Yavorsky refers to the term "modal rhythm... as an unfolding of music in time" (cited in Lupishko, 2016, p. 78), and in the article *Osnovnye elementy muzyki* (1923), Yavorsky "tried to establish a psycho-physiological basis for his theories" (Lupishko, 2016, p. 77).

regardless of whether the central pitch (or pitches) is a constituent (sounded) member of the particular scale in question (see Figure 8).

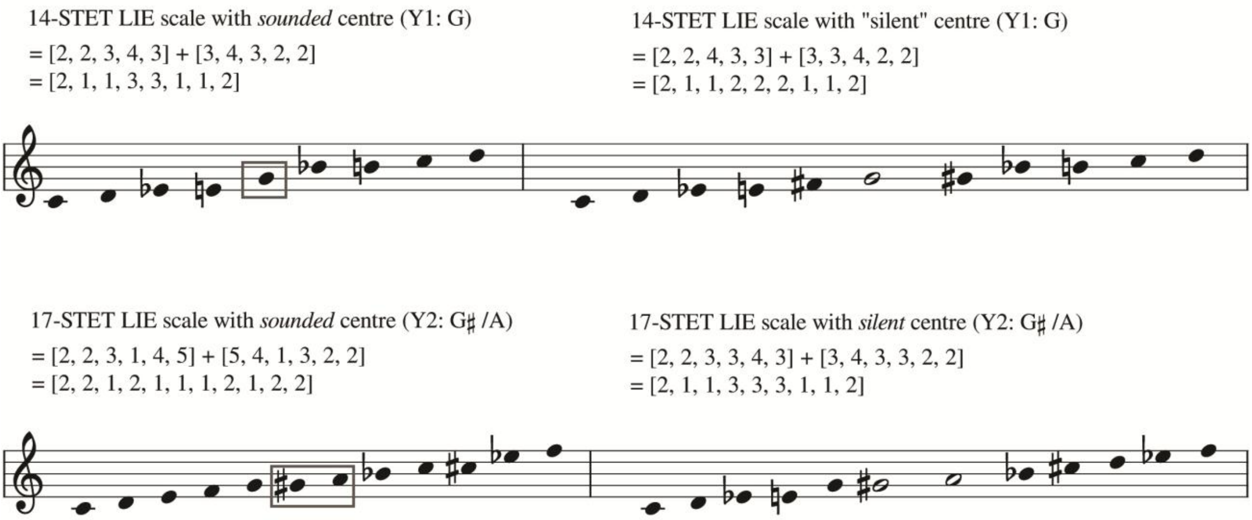


Figure 8: Even [14] and odd [17] axial identities

As mentioned by Ben Johnston, "a group of pitches may be very complexly related to each other, but often all of them can be simply related to another pitch, which need not even be present" (1964, p. 61), and Heinrich Vincent (1894) points out that the "pitch class identified as 0 does not necessarily represent a tonal center… but simply represents a point of reference for any collection of elements" (cited in Nolan, 2003, p. 210). Parncutt and Strasburger suggest that:

> a kind of tonality could be established by defining one sonority, which need not actually appear in the progression, as a kind of tonic, and requiring other sonorities to conform to a certain range of pitch commonality with the tonic sonority. Progressions composed according to restrictions such as these may then be used as raw material for a piece (1994, p. 115).

With LIE constructs, this *kind of* tonic sits in the middle and is defined by its P+I STET endpoints.

(ii) An omnipresent *mese* (axial centre)

The *sounded* Y1 of all *even* centrosymmetric scales is omnipresent when any further (intervallicaly equivalent) nested hierarchies are produced from the original scalic pitches. For demonstrative purposes, the following examples (Figures 9 and 10) use a pentatonic 26-STET scale [6, 7| 7, 6] generated from a central C4 {B2, F3, **C4**, G4, C♯5}, and a 9-pitch 46-STET scale [8, 6, 5, 4 | 4, 5, 6, 8] {C♯2, A2, E♭3, G♯3, **C4**, E4, A4, E♭5, B5}. If the central axial pitch is present (sounded) in the prime scale, when any fixed pitch of any *even* scale is used as a central generative element, all the resulting transpositions will contain the mese from the original prime scale.

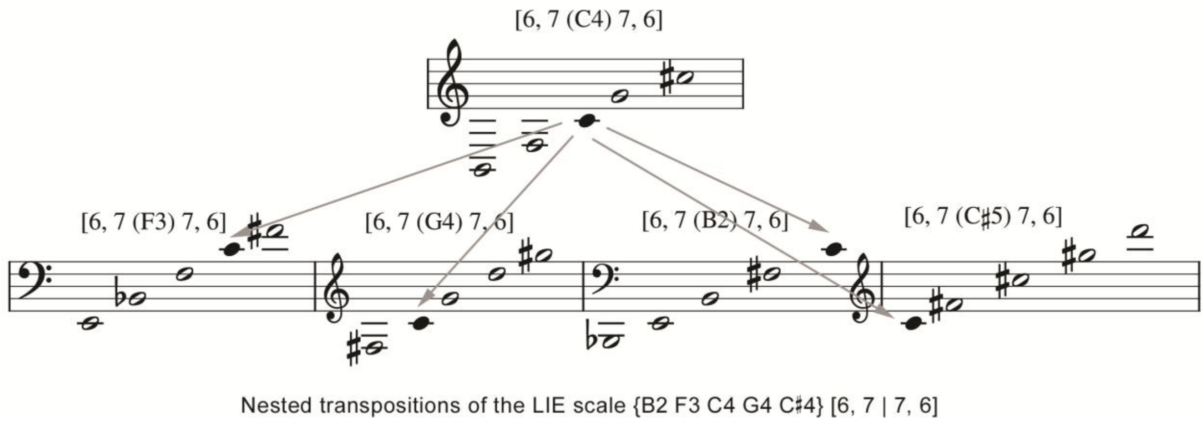
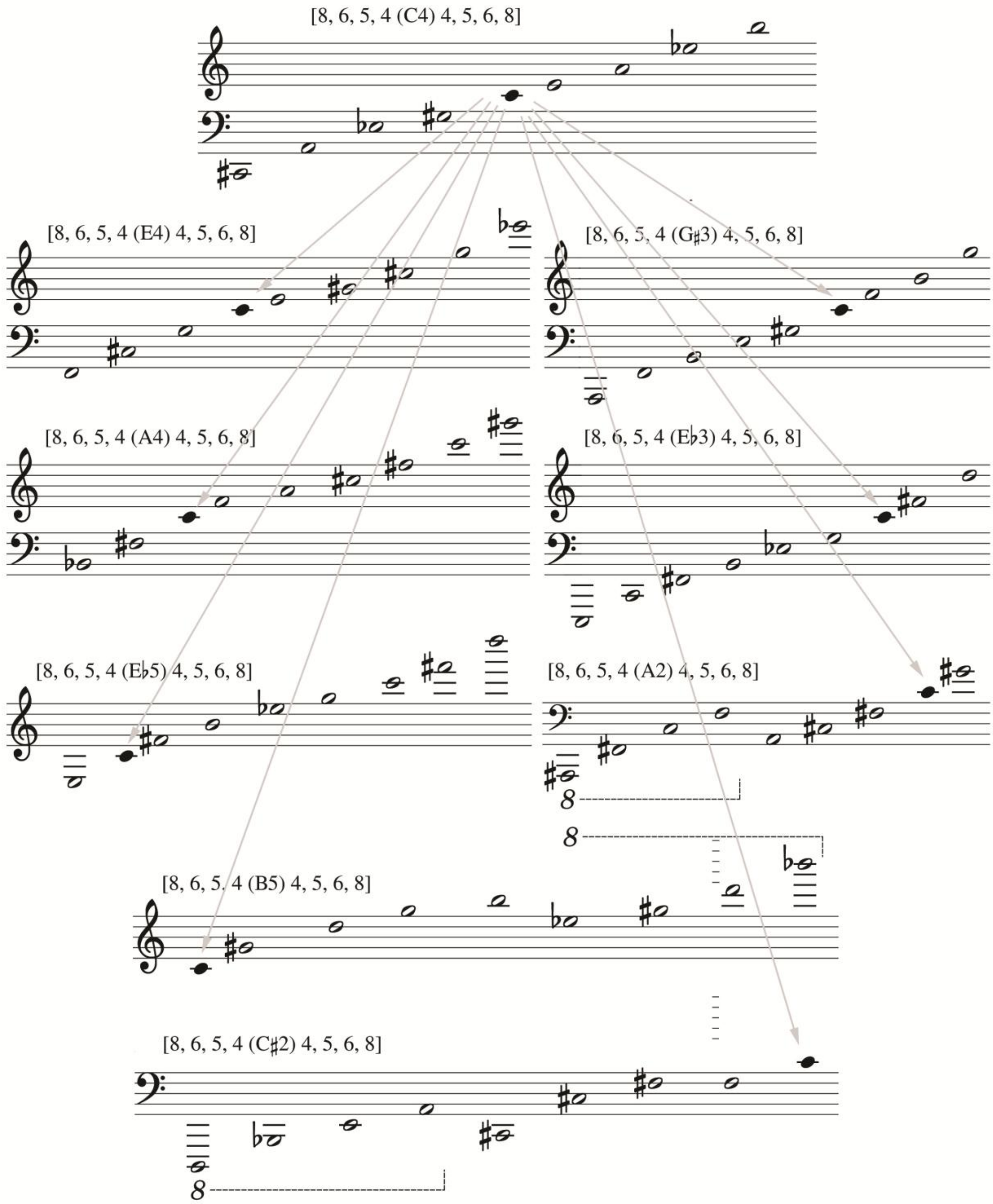


Figure 9: 26-STET example of an omnipresent axial mese



Nested transpositions of the LIE scale {C♯2 A2 E♭3 G♯3 C4 E4 A4 E♭5 B5} [8, 6, 5, 4 | 4, 5, 6, 8]

Figure 10: 46-STET example of an omnipresent axial mese

The axial centre (or *mese*) can now be theoretically construed as an alternative *generative* tonal centre, and although the individual harmonic possibilities within each span/STET are largely reliant on the number (and configuration) of available intervallic partitions/divisors, all P+I scales produce a balanced cardinality for both even and odd collections of any/all spans.

(iii) Balanced pitch cardinality

All the twelve PCs in a single tone row will of course have a cardinality of 1, regardless of sequential ordering. Equally obvious is that all LIE scales with an *even* span around an axial Y1 will unfold compound variants of consonant and dissonant symmetrical pairings, and consequently produce symmetrically balanced dipole cardinalities around the tritone [6]. The following scales (Figure 11) are examples of this.

Figure 11: Examples of Y1 balanced PC cardinality

Balanced cardinality is similarly prevalent with regard to pitch domains of all (Y2) *odd* span P+I scales, albeit with shifted axial balance points; see examples below (Figure 12).

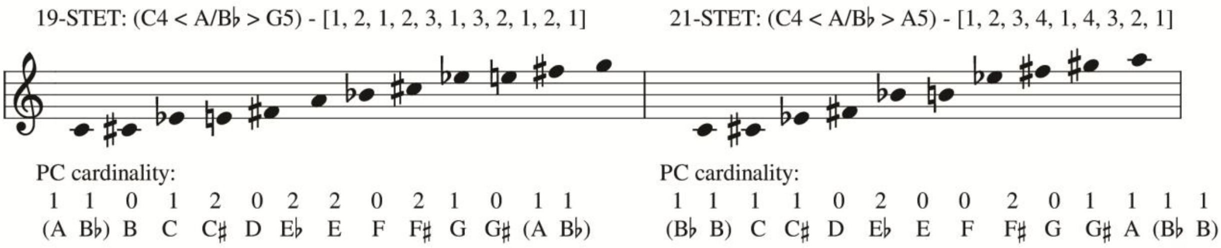


Figure 12: Examples of Y2 balanced PC cardinality

This balanced PC cardinality rule is consistent for all P+I (↑+↓) constructs, regardless of span, and the absence, inclusion or duplication of internal constituent pitches. I have not found any specific 12-tone theory that directly corresponds to this, but the diverse cardinalities and pitch commonalities of various *all-interval* (66-STET) scales are worthy of further discussion.

The all-interval series (AIS) or scale is a twelve-tone row with eleven unique *intervals* (1 to 11), where all the intervals (as well as the 0 – 11 numbered PCs) add up to 66. A much discussed AIS is the "Mallalieu row" {0, 1, 4, 2, 9, 5, 11, 3, 8, 10, 7, 6} (Lewin, 1966, p. 285; Morris and Starr, 1974, p. 384; Alegant and Lofthouse, 2002), the 66-STET ascending intervallic configuration of which is [1, 3, 10, 7, 8, 6, 4, 5, 2, 9, 11]. When rendered as a centrosymmetric P+I LIE (from e.g. A0 to E♭6 in Figure 13, below), the following intervallic pattern emerges; [1, 3, 7, 3, 6, 1, 1, 5, 2, 2, 4, 2, 2, 5, 1, 1, 6, 3, 7, 3, 1].

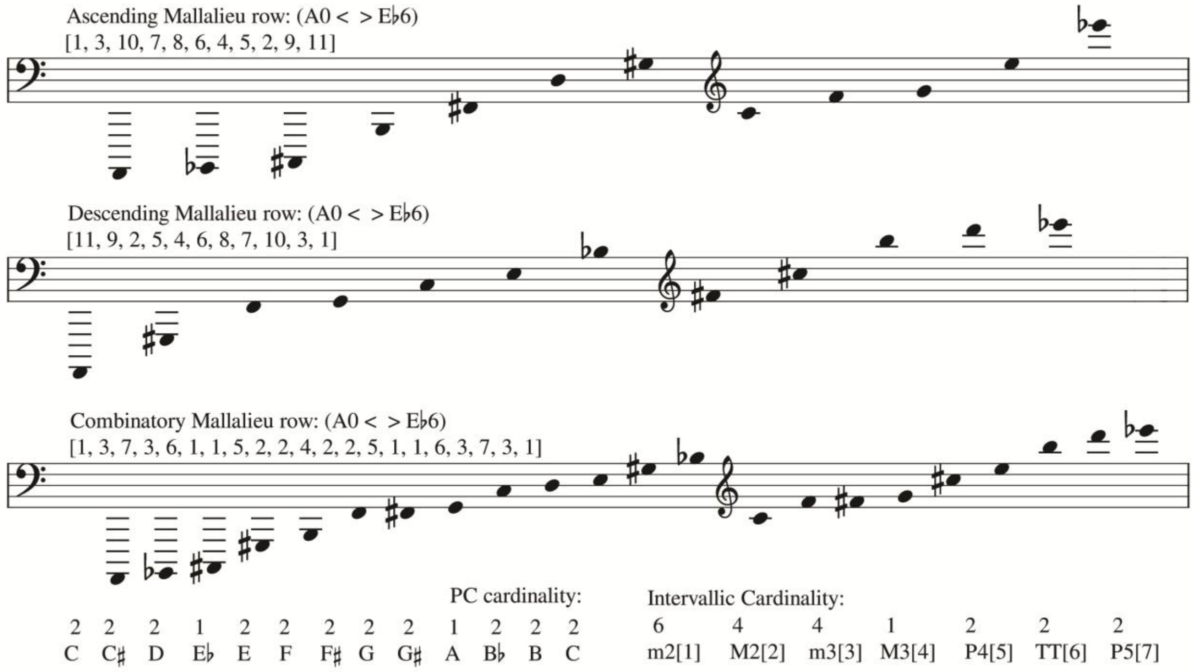


Figure 13: P+I extensions of the "Mallalieu row"

An intriguing structural attribute of this particular scale is that the *intervallic* cardinality (1 to 7) 6, 4, 4, 1, 2, 2, 2, is unique to the "decimal expansion of the sum of reciprocals..." (OEIS: A064890); Appendices I and II of this thesis list various scales that correspond to known (and as yet unnamed) mathematical sequences.

Fritz Klein's "Mother Chord" (Slonimsky, 1947, p. 185, no. 1317) and Slonimsky's "Grandmother Chord" (1947, p. 185, no. 1318) are both AISs with pitch commonalities other than their (±) nadir and vertex points. In the original 'P' versions of both scales, the tritone [6] is located in the middle, and both start with interval [11] and end with [1], yet the "Mother" P+I produces only two internal pitch commonalities, whereas the "Grandmother" has five (see Figure 14, below). Slonimsky achieves this by simply replacing the minor sixth [8] with the major second [2], and the minor seventh [10] with the major third [4].

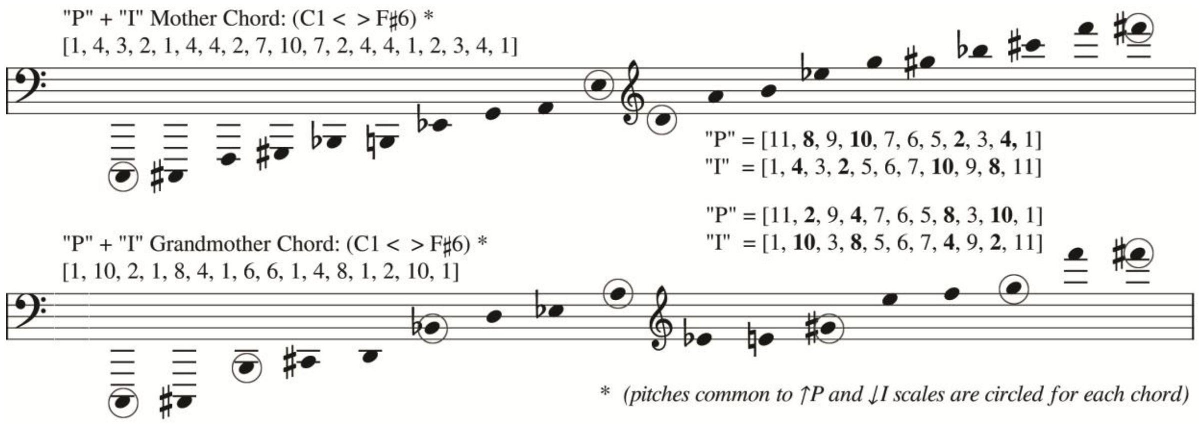


Figure 14: P+I extensions of the "Mother" and "Grandmother" chords

A mathematical theorem that neatly explains these attributes beyond 66-STET (perhaps related to symmetrical and combinatorial partitioning) would be useful, but is beyond the scope of this thesis.[19] However, the potential compositional utility of LIE scales regarding balanced cardinality is of interest. Along with the previously mentioned cumulative counting numbers/mode 2 congruence and the overlapping axial extensions of Bartók's *Piano Concerto No.2* (which are discussed in the commentary for *Libertatia Part III*; in Part 3 of this thesis), discovering the constituent integer/interval elements of Webern's *Piano Variations, No II,* Op.27 (1936) was crucial to better understanding my P+I structural approach beyond major/minor harmonic dualism and subsequent neo-Riemannian theories in general (see Cohn, 1998a).

(iv) Centrosymmetric FPFs used in Webern's Piano Variations, No II

The FPF symmetry of Webern's *Piano Variation, No II* is discussed by (e.g.) Westergaard (1963), Lewin (1993), Nolan (1995), and Moseley (2013). In short, the registral placement of

[19] Within the piano keyboard range alone there are a ridiculously large number of possible P+I configurations, but (not including scales that incorporate semitone [1] intervals) there are only/exactly seven centrosymmetric permutations for all the octaviated (those that add up to 12) combinations of *at least* two different intervals. I.e., [2, 3, 2, 3, 2] (Suspended Pentatonic), [3, 2, 2, 2, 3] (Minor Added Sixth Pentatonic), [2, 4, 4, 2], [4, 2, 2, 4], [2, 2, 4, 2, 2], [3, 6, 3] and [2, 8, 2].

all deployed pitches is structured on centrosymmetric intervallic principles, as is the subset of grace notes [20] (see Figure 15, below).

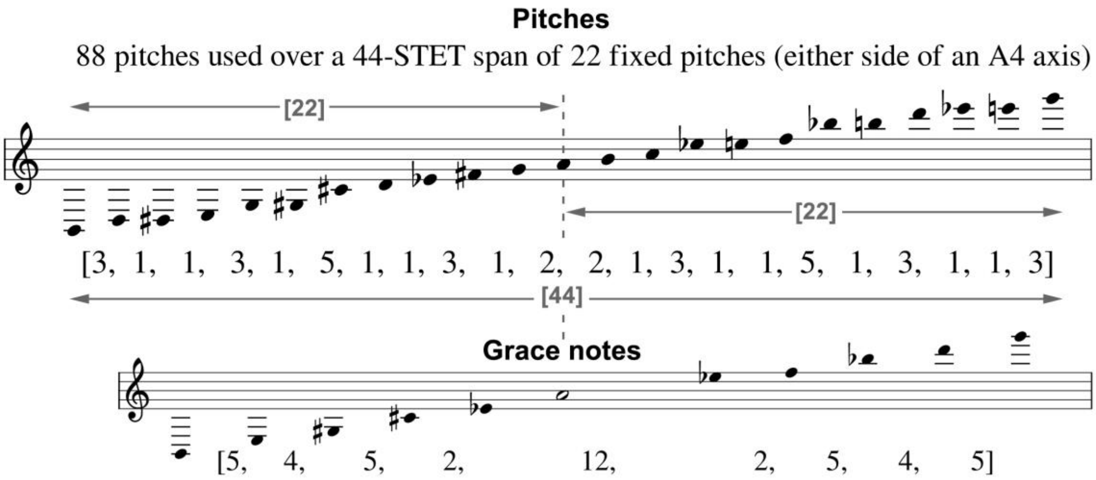


Figure 15: FPFs used in Webern's Piano Variations, No II (Op.27)

Although Nolan suggests that "the row for the Piano Variations does not exhibit the kind of rigid internal symmetries and invariant properties commonly associated with Webern's rows" (1995, p. 50), Lewin mentions that "*some* pitch classes are registrally mobile" (1993, p. 348) and Westergaard suggests that "register for four of the dyads is constant and for the other three more or less so" (1963, p. 114); these comments are perhaps indicative of a general disregard for centrosymmetric and registrally extended FPFs in favour of PCs and row entry order. Through pitch counting it becomes apparent that, in this piece, the complete FPF along with the subsets of *grace notes* and *triads* are all constructed using precise centrosymmetric cardinality (number/amount of elements per set) around an axial A4 pitch (see Figure 16).

[20] The full sequence [2, 1, 3, 1, 1, 5, 1, 3, 1, 1, 3] ascending from the axis does not match any current OEIS sequence, however, [2, 1, 3, 1, 1, 5, 1, 3, 1, 1] is unique to OEIS: A072463, and [1, 3, 1, 1, 5, 1, 3, 1, 1, 3] is unique to A255670.

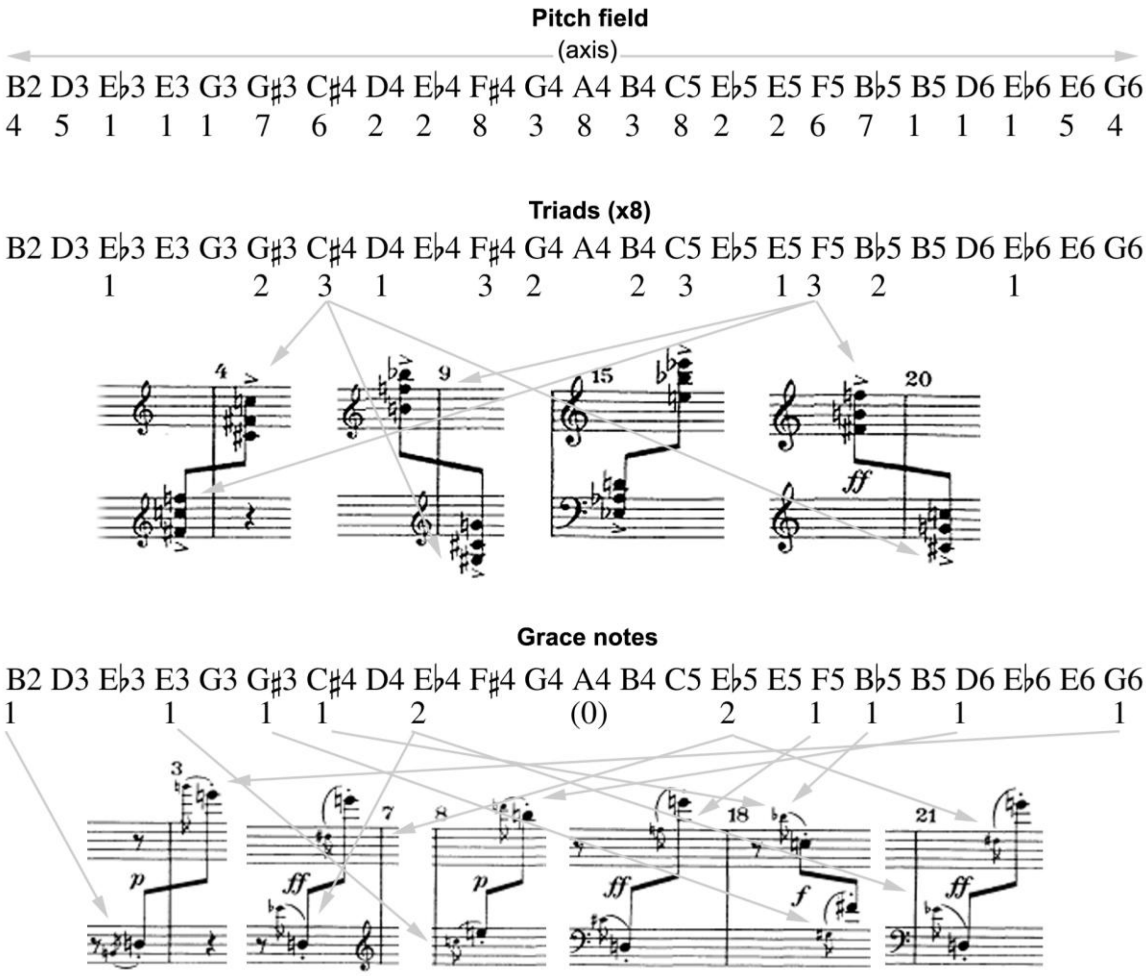


Figure 16: Webern's centrosymmetric pitch cardinality

(v) Palindromic prime (PP)

When collating any pre-compositional LIE scale, as with PC/IC cardinalities, the PP is a useful reference (a kind of auditory *shorthand*) for the amount/quality of OE consonance or dissonance contained within any scale - here are three examples. No.1 is derived from one of Carlton Gamer's 19-TET "Deep Scales",[21] and no. 2 is based on a combination of the much-discussed all interval (1 to 6) tetrachords {0, 1, 4, 6} and {0, 1, 3, 7}. These are mentioned by (e.g.) Gamer (1967a, p. 120), Morris (1995, p. 343), and Amiot (2017, p. 3), and are used by

[21] Carlton Gamer is often credited with defining the term "Deep scale". Gamer however cites Terry Winograd as the source of this expression (see Gamer, 1967a, p. 119).

Carter as a generative source for his second String Quartet (see Bernard, 1993). Example no. 3 is a minimal PP rendering of the quintal "fifth duality" (Hauptmann, 1888, p. 9) relationship.

PP example no.1

Gamer's 19-TET scale (although originally referring to microtonal sub-divisions of the octave 12÷19) = "0 3 5 6 8 11 13 14 16" (1967a, p. 117), and is here expressed as a centrosymmetric 76-STET P+I extended LIE scalic collection (from C1 to E7), as follows:

↑ C1 [+3] E♭1 [+5] G♯1 [+6] D2 [+8] B♭2 [+11] A3 [+13] B♭4 [+14] C6 [+16] E7
↓ C1 [+16] E2 [+14] F♯3 [+13] G4 [+11] F♯5 [+8] D6 [+6] G♯6 [+5] C♯7 [+3] E7

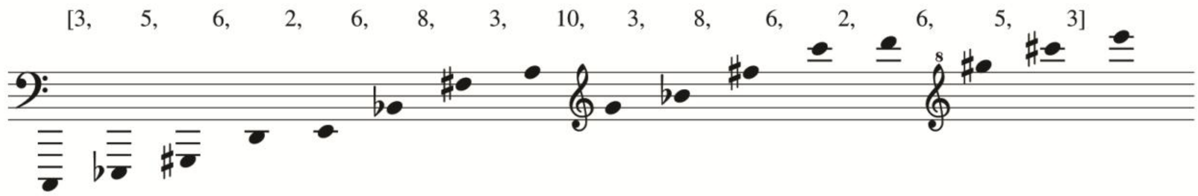


Figure 17: P+I (76-STET) extension of Gamer's 19-TET "Deep Scale"

As the vertices are common, this (9 + 9) combined set is comprised of 16 fixed pitches. The OE set = {C, C♯, D, E♭, E, F♯, G, G♯, A, B♭} and the OG (unused) set = {**F, B**}. [22] In which case, the minimal PP = [1, 1, 1, 1, 2, 1, 1, 1, 1] over a 10-STET span.

[22] Marquis Yi of Zeng's Bell set (*Bianzhong*, excavated in 1978) and the accompanying collection of chime stones, provides evidence for the use of a chromatic scale that pre-dates the European use of carillons and the chromatic keyboard by many centuries. "While traditional Chinese musical theory uses a set of five notes… the Zeng musicians, astonishingly… possessed a set of twelve such notes" (von Falkenhausen, 1992, p.436). The 41 chime stones are all numbered in correspondence with the slots in their three storage boxes and together form an unbroken chromatic sequence that spans 3 octaves plus a major third. Box no. 1 contains stones that fit into an anhemitonic pentatonic scale starting on C (i.e., C, D, E, G, A), Box no. 2 contains the stones that are tuned to a pentatonic scale based on F♯ (i.e., F♯, G♯, B♭, C♯, E♭), and Box no. 3 contains three octaves worth of the missing B's and F's that complete the chromatic (see Bagley, 2005, p. 62). The "label on the lid of box number 3 says something like 'extras'" (ibid, p. 63). The significance of the number of chime stones (41) is not known, but as the bell set was discovered in Hubei, a renowned centre of Taoism, the line "the highest notes are hard to hear" (Laozi et al., 1972, Ch. 41) from verse 41 of the Tao Te Ching is perhaps of interest. Intriguingly, period-41 was the last cellular automata periodic oscillator to be discovered (from 1 to 61) for "Conway's Game of Life" (Brown et al., 2023, p. 1), this was discovered by Nico Brown in 2023 (ibid. p. 18).

PP example no.2

When combined, the all interval tetrachords {0, 1, 4, 6} + {0, 1, 3, 7} = {0, 1, 3, 4, 6, 7}. The PP therefore = [1, 2, 1, 2, 1] over a 7-STET (fifth) span (see Figure 18).

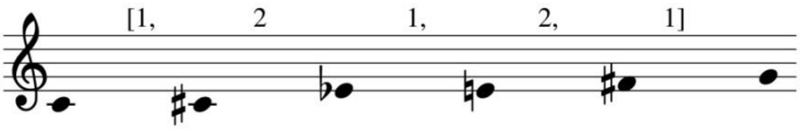


Figure 18: PP of the all interval tetrachords {0, 1, 4, 6} + {0, 1, 3, 7}

PP example no.3

The PP of fourths [5] combined with fifths [7] requires a 35-STET (5 x [7] combined with 7 x [5]) span [5, 2, 3, 4, 1, **5**, 1, 4, 3, 2, 5] in order to produce the first (minimal) isolated incidence of the 12 complete chromatic PCs (see Figure 19). The result is a scale that only differs from Carey and Clampitt's "well-formed" (1989) variant [5, 2, 3, 4, 1, **6**, 1, 4, 3, 2, 5] thanks to a central intervallic fourth [5], rather than tritone [6].

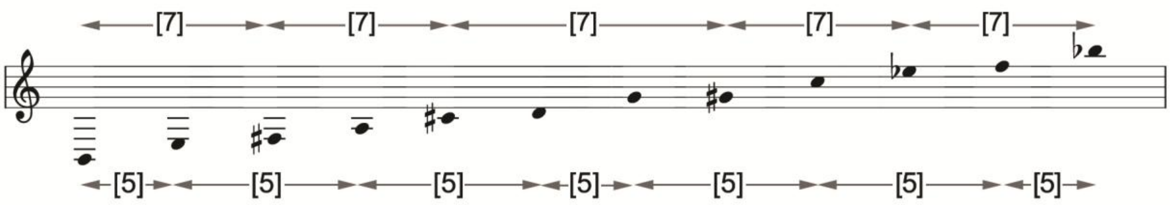


Figure 19: PP (35-STET) B2 - B♭5 "next of kin" scale

In short, the principles (rather than rules) of LIE scalic construction are that FPFs should (ideally) be (i) centrosymmetric, and (ii) registrally extended beyond the 12-STET limit, resulting in an OG collection that may be juxtaposed with its LIE scalic counterpart(s). The caveat to this is that hierarchically *nested* (Xenakis's *metabolae*) and *extended* (I use these terms rather than *transposed*, as transposition, in the traditional sense, is generally not a combinatorial device) LIE and/or OG sets may be utilised if required. In order to explore any alternative harmonic (consonant or dissonant) *weight* and to avoid what might be described as combinatorial implosion (a rapid increase of intervallic distance resulting in a subsequent lack of constituent pitches), meaningful (e.g. bottom, top, and additive inverse) finite constraints/limits need to be put in place, and in various pieces I use the registral limits of the available instrumental forces as a generative limit (see Part 3 of this thesis). This is still in keeping with Lewin's idea of an intervallic vector, "a number (modulo 12) which measures a directed distance from one pitch-class to another" (1977, p. 196), but through registral extension, the auditorily obvious idea that (e.g.) a minor second and minor ninth sound different is explored, $0 + 13 \neq 1$.

Part 2: The perception of P+I scalic constructs

> Music... has only two ingredients... the capacity of a body to vibrate and produce sound and the mechanism of the human ear that registers it... All else in the art of music... is implicit.... Implicit in the man-made part of the musical art are (1) an **attitude** toward one's fellow man and all his works; (2) a **source scale** and (3) a **theory** for its use..."(Harry Partch, 1974, p. xvi – emphasis added).

Regarding Partch's "attitude", LIE scales are architectonic (rather than phrenological) source scales. Perhaps the most familiar phenomenological example of centrosymmetric scalic expansion is the auditory Doppler shift experience,[23] where pitch/frequency is perceived to increase (contract) when the source is moving towards the listener and decrease (expand/dilate) as it moves away.[24] As the two formulas for the Doppler relating to movement '*to* and *from'* a central reference point only differ in the use of "+" or "-", might this be (in some way) theoretically congruent with a perceptual increase or decrease of cumulative intervallic distance, derived from extending pitch intervals from a centralised tonic (of sorts)? Similarly, might numerically extended/dilated (exponential) rather than *set* (proportionally direct, 1 to 1) scales of pitch provide alternative insights into the auditory/temporal experience?

[23] Throughout this research I lived on a wooded hillside in rural Wales, where the predominant sounds (and auditory influences) were birdsong, a flowing stream, and the engine sound of passing (approaching and receding) traffic in the valley below.

[24] However, according to Hall and Moore, "sounds that move towards us have a greater biological salience than those that move away... listeners showed a perceptual bias for auditory looming, judging both the magnitude of the level change and the magnitude of the apparent motion to be greater for the rising than the falling sounds…" (2003, p. 91). From a conceptual aesthetic (*mu-fi* rather than *sci-fi*) standpoint, Lord Rayleigh's observation of the Doppler Effect regarding the velocity of an observer/listener (v) and the velocity of a sound (a) where "If v = 2a, the observer would hear a musical piece in correct time and tune, but backwards" (1896, p. 154), is both informative and inspirational, and must resonate with anyone who has ever spliced tape, manipulated digital audio, or played with a Leslie speaker that uses Doppler related frequency modulation; where relative velocity increases tonal pitch as it rotates towards a listener and lowers pitch as it rotates away, effectively sending sound in opposite directions.

Registral extension and inverse perception >OE

The cognitive recognition of OE is generally thought to be fundamental to musical systems of all cultures, despite the evidence suggested by melodic *Deutsch distribution* (Deutsch, 1972), *stretched tuning* (Jaatinen *et al*., 2019), and traditional scales (e.g. Maqam) that do not achieve octave equivalence at the eighth note. The extent to which OE may be culturally inculcated [25] has recently been addressed by (e.g.) McDermott *et al*. (2016), Pressnitzer and Demany (2019), and Jacoby *et al*. (2019), and regarding pitch *direction*, the Western tendency towards ocularcentric descriptions of the up/down ascent and descent of pitches (for example, the Lark ascending and the decent into Hell) is not necessarily universal. E.g. Brower notes that “Balinese and Javanese musicians refer to ‘high’ and ‘low’ pitches as ‘small’ and ‘large’, while the Suya of the Amazon basin refer to them as ‘young’ and ‘old’” (Brower, 2008, p. 97),[26] and as far as possible we should embrace all of these ways of thinking when considering P+I constructs.

Rameau suggested that chromatic progressions are “more difficult to comprehend, when the Parts descend, than when they ascend” (1779, p. 111), and Burnham *et al*. found that “major modes were identified more accurately when played with ascending pitch, and minor modes were identified better when played with descending pitch” (2021, p. 399), in which case it could be argued that P+I constructs amalgamate or transcend the dualist major/minor dichotomy. However, as our individual (yet increasingly global) culturally

[25] To what extent is our ability to perceive and sing melodies roughly *in tune* across different octaves the result of an evolutionary benefit related to social cohesion (often exploited as a means of social control in the late phase of Capitalism)?

[26] Perhaps the terms *faster* and *slower* are more appropriately applied to registral pitch differences, not least that this is correct regarding physical frequential attributes.

contingent listening preferences cannot be separated from tonality and OE, a sense of up/down harmonic *dualism* continues to permeate this discussion.

Schoenberg believed that the "mind can operate subconsciously with a row of tones, regardless of their direction" (1950, p. 114), and Dowling (1972), Balch (1981), and Krumhansl *et al*. (1987) all found that *inverse* melodic structures were in general easier to perceive than *retrograde* inversions, but this was not explored in relation to extended registral pitch space, or with regard to a generative central point.[27] Similarly, Crowder and Neath (1995) found that consecutive melodic tones "were perceived as having a greater temporal separation if a wide gap in pitch separated the two tones" (1995, p. 379), but did not test this beyond the octave.

Eugene Narmour's "implication-realization model" (1990) suggests that "large intervals at higher levels imply reversal of registral direction" (2000, p. 338), and William Benjamin suggests that "it is the pitch, as opposed to the pitch-class, aspects of twelve-tone pieces that are easiest to take in" (1976, p. 31) and that emergent "registral continuities" (1976, p. 31) are of importance. As mentioned by Paul Lansky, "to observe that a pitch *is* a member of a pitch class is far less useful than to consider *how* it is a member of a pitch class" (1975, p. 31). A utilitarian approach to the FPF >12-tone concept can be derived from Paul Nauert's observation that, "like a chord, a pitch field possesses a characteristic harmonic sonority to which all of its constituent pitches contribute" (2003, p. 181). Lansky suggests that registral

[27] From a drummer's perspective, it is worth mentioning David Dwyer's research into the variously reported phenomena of tonal inversions found in the Loma (South-western Mande) language, which Dwyer attributes to the turning "upside down" (or around), rather than replacement of, an old/worn double headed ceremonial drum skin. For the players, "their visual and motor experiences remained the same" (Dwyer, 1981, p. 441) but in order for the listeners to still "speak the *language* of the drum" (ibid.) with its altered pitch contrasts, linguistic tonal inversions were required.

choice "adds an important dimension to composition by helping to assert relations at a level higher than that of note to note" (Lansky, 1972, p. 41), and in *Pitch-Class Consciousness* (1975), Lansky uses intervallic *expansion* as an example of a *filter*, one that "enables us to attribute properties to relatively abstract relations, [that] ...teaches us to hear" (1975, p. 45); but what are we learning? Is there a phased (scalic or power) metarhythmic[28] relationship between the distribution, perception, and memory of *all* organised sounds?

An interesting justification for the relevance of P+I LIE scales derived from integers is provided by Strait *et al*. (2010), who asked participants (both musicians and non-musicians) to listen to a sequence of numbers, and then to "repeat them in reverse order" (2010, p. 23). The resulting data suggested that musicians demonstrated better "ability to remember more digits in reverse sequence" (2010, p. 24); although the extent to which this was an *attribute* or a *consequence* of (so called) musicality was not addressed. As noted by Zhang *et al*. "it remains unclear how numerosity processing in different modalities interacts within the brain" (2020, p. 164). However, when "asked to place a given number onto its correct spatial position on a horizontal line... older children start to count from the midpoint when the target position is closer to the midpoint than to one of the end-points of the line" (Faulkenberry *et al*., 2018, p. 274). This "midpoint strategy is associated with greater arithmetic competence... [and for adults] ...fixations are distributed along proportional

[28] Antonio Benitez-Rojo suggests that the term metarhythm "can be arrived at through any system of signs, whether it be dance, music, language, text, or body language" (1996, p. 18), and that African culture "cannot do without two words: polyrhythm and metarhythm... each act, each utterance refers in one way or another to a *rhythm-langue* that underlies everything, that precedes everything, that places itself in the very root of all processes and things" (Benitez-Rojo, 1996, p. 170).

reference points (e.g. endpoint, midpoint, points between the endpoint and midpoint)" (ibid.).

Regarding the intrinsic *axial* component of LIE scales, Roitman *et al*. discuss a "bisection point (BP), or value judged as equally similar to the two extreme values" (2007, p. 302), and suggest that a "value that is subjectively midway between the two extreme anchor values can be informative as to the psychological scaling of the subjective number line" (2007, p. 315). [29] Are we similarly tracking/counting consonant and dissonant intervals in relation to a central tritone, or (e.g.) the centrality of an octave within the double octave *systēma* of the ancient Greek *Greater Perfect System* (GPS), as opposed to (or in conjunction with) a fundamental? This relates to Jonathan Harvey's suggestion that with "symmetrical mirroring structures" (1982, p. 2), the bass and our "focal attention is forced into the axial middle, because all relationships converge there: the sounds point to it" (ibid.). Using magnetic resonance imaging (MRI) to create tonotopic maps of the functional organisation of the human primary auditory cortex (PAC), Formisano *et al*. found that "two mirror-symmetric gradients (high-to-low, low-to-high) of best-frequencies can be consistently observed" (2003, p. 862), suggesting that these "primary auditory subdivisions... [are essential to] understanding the cortical organization of the human brain underlying the perception of sound" (2003, p. 866). Crucially, Roitman *et al*. suggest that "subjects may automatically encode both the temporal and numerical attributes of stimuli in some circumstances" (2007, p. 316), and that "number processing may be more automatic than time processing" (2007, p. 314). If these cross-discipline analogies seem spurious, "it is clear from the psychophysics

[29] In this respect, the concepts of a *spectral centroid* (e.g. Ries et al., 2018; Handel and Erickson, 2004) and *a-spatial* (or medial) theories of auditory perception (e.g. Meadows, 2018) are interesting, but beyond the ET chromatic focus of this thesis.

of magnitude estimation… that adults can, at the very least, map relational information from any one dimension onto any other dimension" (Bonn and Cantlon, 2012, p. 12). If nothing else, centrosymmetric P+I LIE scalic constructs provide the scaffolding for exploring the aesthetics of this extended ↑+↓ (superposed) cognitive potential.

As with any *synthetic* scalic constructs, LIE scales need to be thought of (and the compositional outcomes listened to) as *holistic* abstract (12-STET+) entities, perhaps related to a music-specific interval code,[30] rather than registrally fixed OE PC extensions derived from *Inversional Symmetry* (IS) or *Transpositional Combination* (TC) (Cohn, 1988).[31] To a certain extent, although "parsimonious voice leading" (Douthett and Steinbach, 1998, p. 249) and fixed "common tones" (ibid., p. 243) are useful tools for composing with *any* scalic or chordal progressions, when dealing with LIE scales, the application of both diatonic theory and traditional 12-tone PC set and group theoretical concepts may be ostensibly null and void beyond the 12-tone limit.[32] I.e. Major triads produce two minor triads as mediants (and vice-versa), but only through symmetrical extension is it revealed that an 80-STET (e.g. B♭0 to F♯7) frame of reference is required to evenly map out the chromatic (two incidents of each PC) from combining the mediants ↑iii + ↓iii, which from 'C4' = {A3, C4, E4, G4, B4} [3, 4, 3, 4]. This becomes {B♭0, C♯1, F1, G♯1, C2, E♭2, G2, B♭2, D3, F3 | A3, C4, E4, G4, B4 | D5,

[30] If we suggest that ± (so called) "melodic" contours are intrinsic to centrosymmetric structures, then a cortical processing justification for this might be related to what Laurel Trainor *et al*. describe as a "new type of information" in relation to "relative pitch distance or pitch interval" (2002, p. 439). Trainor *et al*. suggest that "most work on how pitch is encoded in the auditory cortex has focused on tonotopic (absolute) pitch maps... [whereas] melodic information is thought to be encoded in the brain in two different 'relative pitch' forms, a domain-general contour code (up/down pattern of pitch changes) and a music-specific interval code (exact pitch distances between notes)" (2002, p. 430). Trainor *et al*'s results imply that there are "also cortical circuits encoding interval information independent of the absolute pitch information" (2002, p. 439).

[31] Although both analysis and composition are concerned with function, the analyst is perhaps more interested in the differences between IS and TC, whereas the composer seeks to juxtapose, integrate, fracture or exaggerate any/all contrasting auditory possibilities.

[32] This similarly applies to the potential relevance of the neo-Riemannian *Tonnetz* (Cohn, 1998b).

F#5, A5, C♯6, E6, G♯6, B7, E♭7, F♯7} [3, 4, 3, 4, 3, 4, 3, 4, 3, 4 | 3, 4, 3, 4 | 3, 4, 3, 4, 3, 4, 3, 4, 3] and places a new (Y1) axis at D4; see the 80-STET entry in Appendix II of this thesis.

Perhaps an extended syntactic alternative to *Dissonant Counterpoint* [33] (Cowell, 1930; Seeger, 1930; Rao, 2001; Spilker, 2011), and/or Lerdahl and Jackendoff's *Generative Theory* (1983) is required in order to elucidate compound (12-STET+) harmonic functionality and to re-evaluate extended connotations of tonal consonance/dissonance, which, traditionally, is a (potentially embodied) [34] hierarchical centrosymmetric construct that we experience, measure (or count) from the centre outwards (see Figure 20, below).

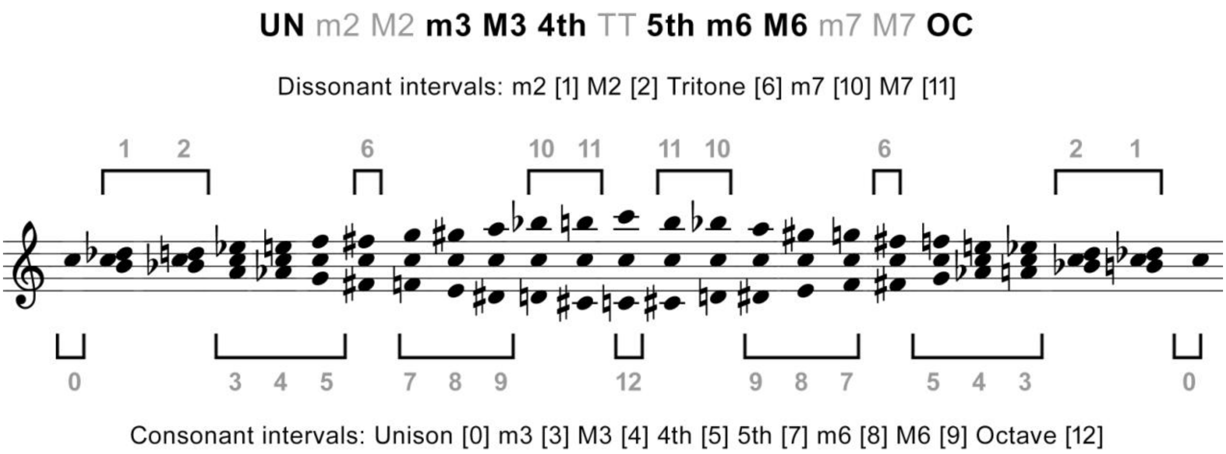


Figure 20: Consonant/Dissonant Symmetry

[33] Dissonant Counterpoint (DC) is now generally accredited to the combined work of Henry Cowell, Ruth Crawford and Charles Seeger (see Spilker, 2011, p. 482). Seeger suggests a system that inverts the traditional rules of dissonance and consonance, where the "octave, fifth, fourth, thirds, and sixths were regarded as consonant and the tritone, seconds, sevenths and ninths, as dissonant" (Seeger, 1930, p. 26). Similarly, for Cowell, the minor seconds, major sevenths, and ninths were the *foundation* intervals, whereas the major seconds, tritones, and minor sevenths "might be used as alternatives; all thirds, fourths, fifths would only be permitted as passing or auxiliary notes. Octaves would be so far removed from the fundamental intervals in such a system... and might not be used except in the rarest circumstances" (Cowell, 1930, p. 39).

[34] Bidelman and Grall suggest that "elements of music are mapped within early cerebral structures according to higher-order perceptual principles and the rules of Western harmony rather than simple acoustic attributes" (2014, p. 204).

All modes are potentially symmetrical

With a re-appraisal of both the (up/down) tertian and 12-tone chroma limit, we can theoretically do anything we like with scalic material and harmony by addressing the meaning of centrality, and a starting point, within an extended span/STET. If we think of ET semitonal pitch successions beyond (rather than within) their OE sets/partitions, then both of the common (major and minor) pentatonic scales ICs [2, 2, 3, 2 ,3] and [3, 2, 2, 3, 2] can be heard (cognitively appreciated) as palindromic heptatonic scales, if the major scale occupies a 16-(rather than 12)STET set [2, 2, 3, **2,** 3, 2, 2], and the minor is placed within a 17-STET (eleventh) frame of reference [3, 2, 2, **3,** 2, 2, 3] (see Appendix II of this thesis).

Unlike Pythagorean tradition, Ptolemy, Boetheius, and the author of the *Scolica Enchiriadis* (SE), all treat the eleventh as a consonant interval (Barbera, 1984, pp. 202–204). In Medieval modal theory, authentic modes were known to have an ambitus (span) up to a tenth above the final (Powers *et al*, 2001), and the ambitus of a plagal mode might include a sixth above, and a fifth below the final (Powers, 2001).

When expressed as centrosymmetric structures, the seven modern modes reveal interesting attributes regarding span and length spectra (see Figure 21 below; W = whole tone, H = half). [35] It is well known that the traditional modes exhibit intervallic mirrored equivalence; Lydian (↑) = Locrian (↓), Ionian (↑) = Phrygian (↓), Mixolydian (↑) = Aeolian (↓), but it is worth

[35] Wai Yan Pong's Conjecture states that “a natural number is a sum of consecutive integers if and only if it is not a power of 2” (2009, p. 1). None of the symmetrical STETs/spans or length spectra of the seven modes add up to a power of 2, and only the Dorian is an octaviated construct.

reiterating that only the Dorian mode is centrosymmetric within 12-STET (octaviated) spans.[36]

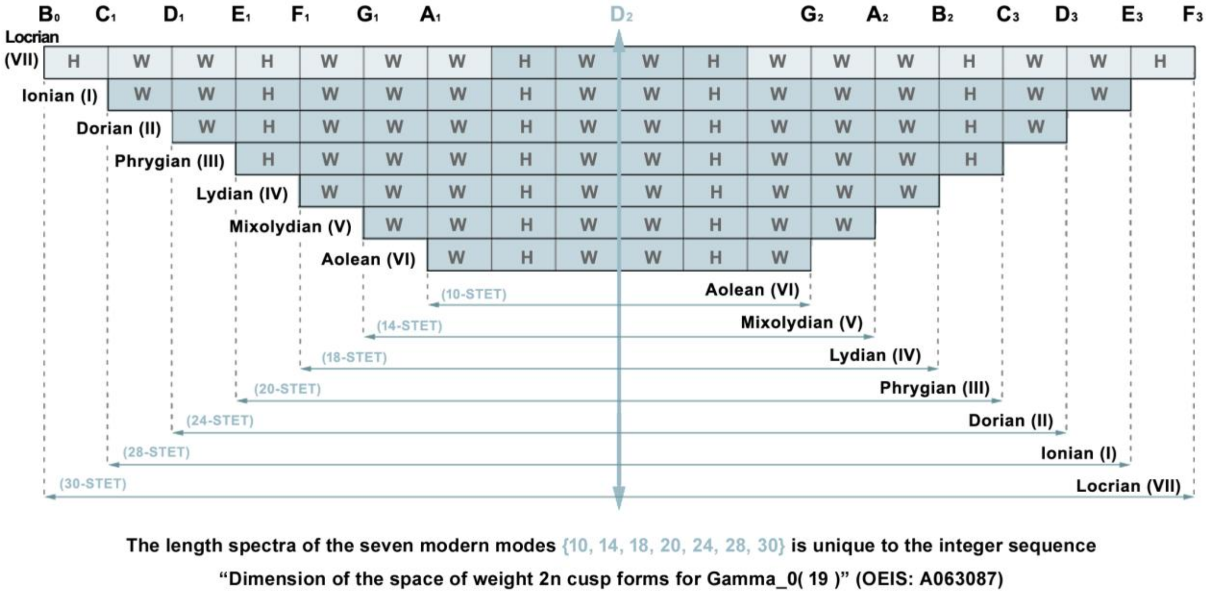


Figure 21: Centrosymmetric modal extensions

Once we break *out of* (rather than microtonally *into*) 12-STET partitioning, the question becomes, how large, how many, and what kind of cycles sets or collections are we talking about and listening to?

The "fifth duality" and "well-formed" cumulative scales

If LIE scales can be described as a "*proto-tonal theory*... a theory of the *background system* of tones prior to the selection of one particular tone as referential" (Agmon, 2013, p. 104),[37] then intrinsic to this is the *directional* ambiguity of the cycle of fifths/fourths. Hauptmann

[36] Although chroma in relation to visual colour is not here explored, as the visual tone *gray* is similarly central to the monochromatic black-white range, one might wonder if Oscar Wilde was alluding to this bidirectional polarity when choosing the name *Dorian Gray* (Wilde, 1891).

[37] This differs from Andrei Miasoedov's definition of *proto-harmony* (pragarmoniia) from his *Garmoniia russkoimuzyki* (Harmony of Russian Music), which (at the time of writing) remains untranslated into English, as discussed by Ellen Bakulina (2018).

calls this the "fifth duality" (1888, p. 9), Alois Hába calls it a "double inversion" (cited in Battan, 1980, Part II, p. 108), [38] Schoenberg refers to it in terms of "next of kin" (1911, p. 24) and opposing *gravitational* forces (ibid.), Westerby uses the term "*pons asinorum*" (1902, p. 32) and Hanson suggests that if the 12-tone and 11-tone scales are "actually only one scale form" (1960, p. 356), then the fourth and fifth are the same interval, which he called "P" for perfect.[39]

Both the ascending (+7) and descending (-5) cycle of fifths/fourths (from 'C') produce the ordered PC set {C, G, D, A, E, B, F♯, C♯, G♯, E♭, B♭, F...}. However, to a certain extent, Diletskii's familiar circle of fifths [40] is a static (non-temporal and non-registral) ocularcentric misnomer, in that a 60-STET (5 octave) span is required to compete the circle (or chain) of fourths, and an 84-STET (7 octave) span completes the fifths. [41] Crickmore discusses "the inverse proportional relationship between position and pitch [and] warns the reader of the need to look beyond appearances towards an understanding of the idea of reciprocity" (2006, p. 22). Of course the circle of major/minor keys that Busoni refers to as the "bifurcated garment" (1911, p. 27) where the "four-and-twenty keys are simply an elevenfold transposition of the original twain" (ibid.) is an OE construct, but if we ignore

[38] Although Hába is perhaps best known for his microtonal work , taking Battan's (1980) English translation of Hába's Neue Harmonielehre (1927) as a referential source , almost a quarter of Hába's 400 page treatise on his systemic theories is dedicated to the discussion of extended and symmetrical scales, in various guises and configurations.

[39] For an interesting registral unfolding of the "next of kin" fifth duality that omits only the PCs F and G (from C), see the "108-STET (23-pitch) extension of the Virahanka/Fibonacci (mod 12) sequence" in Appendix II of this thesis.

[40] According to Jensen, "the earliest circles of fifths appear in a Russian-language treatise on composition, Nikolai Diletskii's *Grammatika* (Grammar), written in the late 1670s" (1992, p. 331). Alternatively, see Daniélou's chapter on "The Cycle of Fifths" (1945, p. 58) pertaining to the Chinese and Hindu traditions.

[41] Crocker points out that "in some sense the fourth itself is a limma, left over from the projection of the fifth back into the octave. The fifth, in its own way, is a limma, left over from the projection of the octave forward into the twelfth (1:3). Only the octave seems to remain aloof from this process, being generated in some more mysterious way directly from the womb of unity itself" (1963, p. 197).

Hindemith’s suggestion that “a system without the octave… is conceivable, but it would be too clumsy for practical use” (1945, p. 25),[42] the fourth/fifth duality can be simply enumerated as a registrally extended 84-STET P+I construct from (e.g.) C1 to C8 with an F♯4 axis of symmetry (see Figure 22, below).

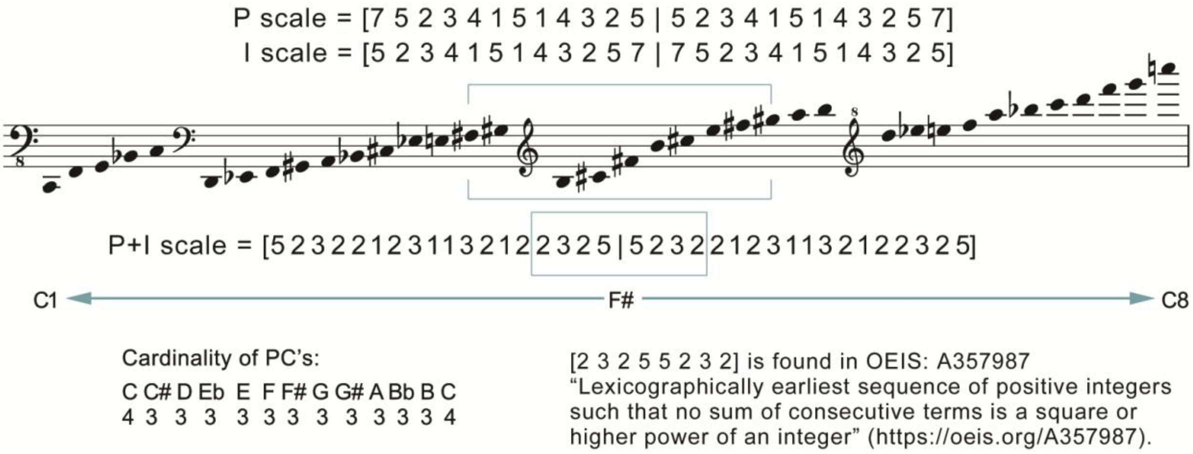


Figure 22: C1 - C8 (scale of fifths) + C2 - C7 (scale of fourths)

The (predominantly circular) topological (Eulerian and Riemannian) *Tonnetz* models of musical distance, suggested by (e.g.) Safi al-Din al-Urmawi (1252), Partch (1974), Lewin (1987), and Toussaint (2005) are, to a certain extent, ocularcentric. Tymoczko defines musical distance as “inverse *similarity*” (2009, p. 258), but arguably the maths behind all *lattices* and *Tonnetz* describes a visually hegemonic OE (but not necessarily registrally relevant) representation of transposition. Although we can picture/imagine the diagrammatic key signature circle of fifths with our eyes closed, it is only through listening to intervallic distances that we perceive the harmonic relationships.

[42] Tristan Murail similarly suggests that “non-octave space [is] arbitrary” (2005, p. 138), however, see the 66-STET (16-pitch) adaptation of Murail’s ‘C’ dynamics in Appendix II of this thesis.

Taking the smallest distance between pitches, which Carey and Clampitt refer to as the "normal form of interval class" (1989, pp. 192–193), the intervallic sequence produced (in relation to their generative 'F' pitch) is the centrosymmetric IC pattern [5, 2, 3, 4, 1, 6, 1, 4, 3, 2, 5]. F → C = 5, F → G = 2, F → D =3 etc (see Figure 23, below).

Figure 23: Normal form of interval classes, based on Carey and Clampitt, 1989, p. 193

It is this sequence of intervals from which Carey and Clampitt define the *primary* and *non-primary* (positive and negative) intervals of their "Well-formed Scales" (1989, p. 193). By taking OE and the fifth as a generative source, and applying both *closure* and *symmetrical* conditions, Carey and Clampitt conclude that "in a well-formed scale... the notion of a generic interval is consistent with, indeed equivalent to, scale-step interval measure" (1989, p. 201), and their 1989 paper does indeed provide "a greater awareness of the way pitch is organized and of the way the mind organizes musical pitch" (1989, p. 206). However, only when rendered as a cumulative P+I LIE scale does this intervallic sequence [5, 2, 3, 4, 1, 6, 1, 4, 3, 2, 5] produce the OE heptatonic pitches of the centrosymmetric Dorian Scale [2, 1, 2, 2, 2, 1, 2], with a silent central axial point at 'F♯' (from 'C'). When this axial point is *sounded*, the pitches of the Blues Scale II [2, 1, 2, 1|1, 2, 1, 2] [43] are produced (see Figure 24, below).

[43] See Ring (2016, scale 1773).

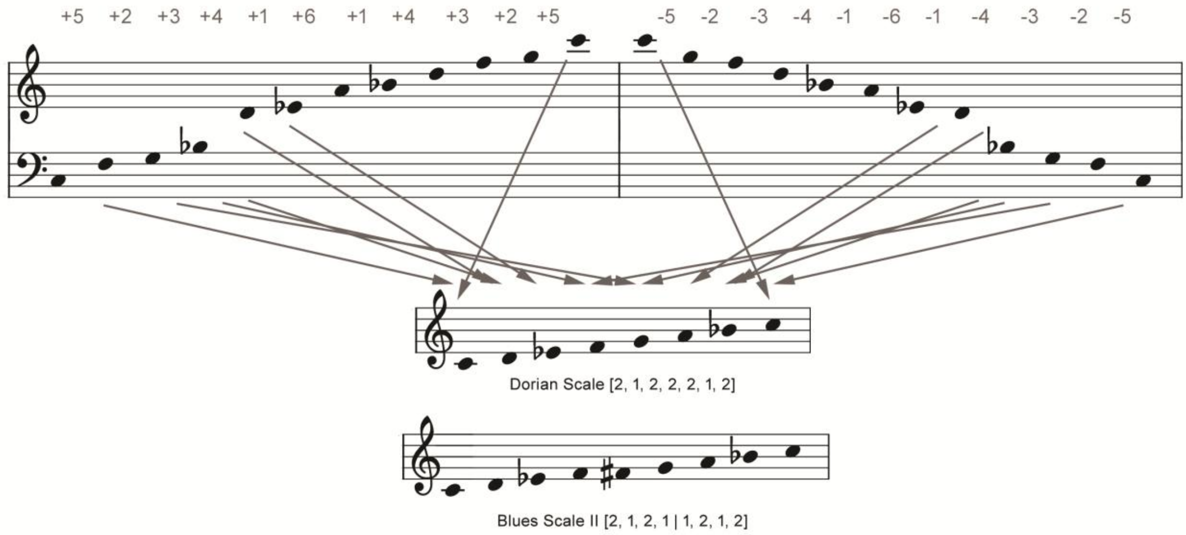


Figure 24: P+I LIE scalic rendering of Carey and Clampitt's "well-formed scale"

An important underlying consideration is the extent to which the perceptual stasis from flux attributed to the OE tonic/dominant "I V I" concept of tonal resolution (from which rock n' roll's "I IV V" is partly derived) is reliant on our cognitive appreciation of the omnipresent ambiguity of the 10 and/or 14-STET "next of kin" (e.g. {G3, C4, F4} [5, 5] and {F3, C4, G4} [7, 7]) relationship, which Lewin refers to as "inversional balance" (1968, p. 1).

Lewin's *transformational attitude* suggests the importance of movement *within* intervallic space, rather than the categorization of the resultant PCs. Lewin, however, asks us "how far we might carry the notion of a direct inversional perception of a relation between pitch-classes" (1977, p. 234), and argues that "such inversional relations, latent if not manifest, should be observable in a large variety of compositional situations, on the general aesthetic principle that 'what goes up must come down' in a variety of senses in various musical idioms" (1977, p. 234), but then decides to "personally not carry this argument so far" (1977, p. 235). It seems that when harmonic dualism, mirroring, reflective or utonal structures are under discussion, many commentators run for the hills, and Lewin remarkably concludes the above essay by stating that:

> I am not sure to what extent I personally subscribe to the transformational, rather than traditional intervallic approach to the conceptual issues under discussion here, beyond an appreciation of the methodological virtues of the formalities for some purposes. I am convinced, at any rate, that there is a definite argument to be made for such an approach (Lewin, 1977, p. 235).

LIE scales are perhaps one such argument, and here we are firmly planted between the hills, listening to the centrosymmetric echoes bounce back at us from either side, then recede at a steady tempo.[44] The 60-STET (circle of fourths) and 84-STET (circle of fifths) of course produce identical PCs, but an interesting symmetrical extension of the consonant/dissonant model is that the combined (60 + 84) 144-STET (12 x 12) of *all* centrosymmetric octaviated scales produce common (although unique to each scalic configuration) pitches when expanded by the paired (↑↓) factors of 1 (12-STET) and 11 (132-STET), 2 (24-STET) and 10 (120-STET), 3 (36-STET) and 9 (108-STET), 4 (48-STET) and 8 (96-STET), 5 (60-STET) and 7 (84-STET), see Figure 25, below.[45]

[44] Extended numerical attributes of Lewin's inversional balance might be explained through a feedback loop between the *in vivo* (wired in, perhaps unconscious or subliminal) experiential analysis (measurement) of the temporal distance between auditory objects in the *in vitro* auditory scene. Beyond the scope of this thesis is the extent to which this alludes to a numerical, granular, patternist, atomist and mathematically (countable) substrate or matrix, where these frequential substrates might be related to (e.g.) Max Tegmark's "Mathematical Universe Hypothesis (MUH)" (2008), J.A. Wheeler's "it from bit… information-theoretic… participatory universe" (Wheeler, 1990, p. 311) and/or Julian Barbour's "Bit from It" (2011), and perhaps Hameroff and Penrose's Orch OR theory (that relates "beat frequencies" to "conscious moments") (2014, p. 57). These examples suggest that musical (frequential/countable) patterns/oscillations might inform what David Chalmers calls the "hard problem" (1997, p.xii) of consciousness; although I disagree with Chalmers' suggestion that a "unified qualitative experience arises from a chord, but not from randomly selected notes" (1997, pp. 7–8), as any collection of notes constitutes a chord, the qualitative nature of which is dependent on the context.

[45] Of the 72 numbered Melakarta (or Mela) ragas only six are centrosymmetric, and these play increasingly prominent roles as compositional source scales (see Part 3 of this thesis).

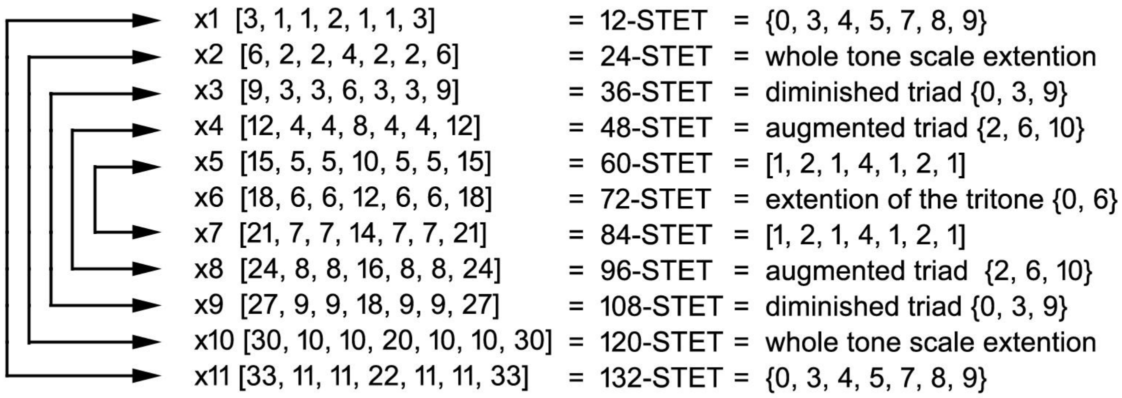


Figure 25: Octaviated expansions of the 31st Melakarta raga [3, 1, 1, 2, 1, 1, 3]

Through registral extension (both octaviated and non-octaviated), P+I LIE scales suggest a self-contained and balanced harmonic template.

Justifications for the use of ET

Although LIE scalic principles might be applied to any *fixed* tuning system (not least quarter tones; a possible area of future research), for primarily practical compositional reasons I currently use the semitone as a lowest (or smallest) common denominator. This is in keeping with Arsistoxenus' "protos chronos", the smallest useful division of sound, but here this refers to 1/12 of an octave rather than 1/12 of a tone.[46] Although ET tuning is clearly a compromise (Halewood, 2015), for both Hauer and Schoenberg it was an aesthetic preference (Yasser and Schoenberg, 1953, p. 60), and if Debussy "was content to incorporate the overtone series into his music through the lens of equal temperament... without requiring a literal (i.e. just intonation) realization of those sonorities" (Don, 2001, p. 69), then ET is a good enough realm to explore.

[46] Is a 72-STET (octave x 6) extended chromatic progression any less relative than Arsistoxenus' 1/72 (semitone ÷ 6) division of an octave?

Utilising ET is also a means of avoiding (i) the rabbit hole of *Just* and *Mean* (*et al*.) temperament and the various 5, 7, 15, 17, **19**, 22, 23, 27, 29, 31, 34, 36, 41, 46, 53, 72 and 96-TET (or Edo) tuning systems from (in the West) Aristoxenus onwards, and (ii) the elitist, musician vs. non-musician, perception of such (Bailes *et al*., 2015).[47] Although a refined awareness of all intervallic relationships can (and should) be learned, all systems are abstractions, a compromise, and variously come up against the Pythagorean comma or limma, from which we derive the whole-tone (8:9 ratio). ET is however contemporarily heterarchical, and even Partch notes that when twelve-tone ET is perfectly tuned "the very least falsity that is involved in its representation of small-number ratios is the discrepant 2 cents in '3/2' and '4/3', which is considered even by most advocates of just tuning as *inconsequential*" (1974, p. 434).

[47] Freya Bailes *et al*. found that "musical expertise facilitates the sensory and perceptual discrimination of microtonal intervals from equal-tempered intervals… while non-musicians appear to have categorically perceived microtonal intervals as instances of their neighbouring equal-tempered counterparts" (2015, p. 5), also that "microtonal intervals rated as significantly more rough and less liked than consonant intervals" (ibid.) and that a stronger measure of pre-conscious evoked response potentials (ERP) was found for equal-tempered rather than microtonal stimuli.

Part 3: Commentary on the development of compositions

Aesthetic influences and methodology

> If the essential quality of a work of art could be described adequately in language then the work of art would be nothing but an illustration of the text, and thus stop to be an independent work of art... Meaningful information seems to exist in the 'sweet spot' between order and chaos (Adriaans, 2009, p. 653).
>
> To compose is... to locate liberation not in a faraway future, either sacred or material, but in the present, in production and in one's own enjoyment (Attali *et al*., 1985, p. 143).

The relationship between extended scalic structures and compositional functions perhaps lies somewhere between (i) Claude Levi-Strauss's description of the relationship between content, structure and form, where "content derives its reality from its structure, and what is called form is the 'structuring' of local structures, which are called content" (cited in Boulez, 1985, p. 90), and (ii) what Hendrik Hofmeyr (an occasional protagonist of symmetrical scales) calls "the art of concealing art" (Hofmeyr cited in May, 2017, p. 29), where "structure fulfils the same role in music as the skeleton does in the human body: a beautiful and functional body would be impossible without it, but it is not the immediate focus of our appreciation" (ibid.). The compositional methodology I employ is thus simply to (i) define and collate extended scales of interest (the shape of the skeleton), then (ii) explore any musical syntactic potential through improvisation and, if worthy of doing so, (iii) commit to an outcome/score.

All of my recent *scored* work is intended for live performance, and much compositional time is spent on developing and best articulating an occurrent (potentially adrenalin-fuelled) interaction/outcome; which, after participating in many thousands of cross-genre live performances (across 4 continents), I suspect will work.[48] Since the 1970s I have been publically performing original music (mostly songs) and have continued to work with a plethora of diverse musicians but, as none of my mature (post-MA, 1999–2024) *scored* compositions have been performed in public in the UK (outside of academic workshops and recitals), and as I could only afford to have a certain amount of my miniatures recorded, in the accompanying portfolio, various virtual renditions necessarily provide proof of concept. Although the viscerality of live performance is a paramount compositional consideration, and the potential *power* of "participatory discrepancies" (Keil, 1987) is ostensibly neglected, no single performance is ever a definitive article. The *quality* of any performance is dependent on the skills and engagement of the individual(s) involved.

Regardless of available performance opportunities, the joy (if not purpose) of instrumental composing is to explore the retentional aspects of sound, by juxtaposing the movement of rhythmical pulses with synchronic (architectonic and harmonic) representations of pitch. Within this (Brentano's) "Unity of Time-Consciousness (UTC): [where] our experience of succession is a unitary phenomenon and not a succession of experiences" (Fréchette, 2017, p. 4) we are (perhaps unavoidably) seduced by melody, but to what extent might melodic (note entry order) concerns conceal the foreplay of countability? The only pre-compositional parameters (as such) here deployed are; (i), the music is predominantly *wordless* (devoid of any associative emotive linguistic syntax, rather than connotations of "pure" or "absolute"),

[48] To use a sporting analogy, a live performance without an audience is the equivalent of a training game. Live audiences are the best (if not the only) arbitrators of what constitutes a "successful" performance. By monitoring audience responses, performers learn what works, and why. As a session drummer with many decades of experience, and a successful street entertainer for the best part of ten years, the number of people I have personally performed to probably exceeded a million a while ago, and when one is doing this for a living, gauging audience response becomes instinctual.

(ii) LIE scales are the primary reservoir of tonal resources, (iii) melody is often an *a posteriori* (Deutsch distribution-esque) consequence of an intrinsic klangfarben registral unfolding of the scale(s) involved, and (iv) all music is written to be played over a strict temporal framework; a kit-drummer's raison d'être (see Bizzell-Browning, 2024). Melody and timbre thus become emergent entities rather than structural prerequisites.

The intervallic arrangement of various P+I LIE scale collections are here explored through trial and error at the piano, and a lot of listening, in order to discover why particular scalic configurations sound the way they do. As mentioned by Lach Lau, harmony "must be *assembled*... discovered... it cannot be purely deduced, but needs to have an empirical axiom [Hume's experiential *Copy Principle*] to get started... this produces something which is qualitatively different from the mere aggregation of its components" (2012, p. 131). The degree to which the outcomes might be regarded as successful is largely dependent on the extent to which we appreciate (or at least acknowledge) the aesthetic potential of extended abstract harmonic polarity, and the relevance of a non-octaviated (*n*-tone), chroma and register-relative concept of symmetrical centrality. Aesthetically, the sweet spot between structure, form, function, and outcome is here creatively explored through improvisation and experimentation, a balance between what Richard Taruskin calls the "incomprehensible results of unknowable plans" (2009, p. 26), and freedom of expression. I conceptually aspire to the percussive piano works of (among many others) Cecil Taylor [49] and Leo Ornstein, [50] and regard the recent orchestral work of (e.g.) James Lee III (i.e., *Shades of Unbroken Dreams*, 2023) and Gavin Higgins (i.e., *Velocity*, 2014) as contemporary large-scale

[49] The Cecil Taylor Unit *Live in Paris* (1969) (i32504, 2021).
[50] Leo Ornstein's *Wild Men's Dance* (*Danse Sauvage*) (1913) (Ian T, 2014).

counterpoint benchmarks. By chance rather than specific intent, some of my pieces for soloists and smaller ensembles perhaps bear a resemblance to Henry Threadgill's. [51]

Commentaries

Music for player piano (Nos. 1, 3, and 6) (2017)

Duration: 9' 39"

The idea behind these pieces was to produce piano roll outcomes. I was reliably informed that *real* piano rolls (physical cylinders) could be created using two MIDI files, hence why these pieces are scored for two piano parts. [52]

With hindsight, *No. 1* and *No. 3* desperately cling to the 12-tone world. *No. 1* employs the FPF {B, C, F♯) [1, 6, 5] throughout, only tentatively alluding to the possibility of a centrosymmetric intervallic structure [1, 5, 1] with the inclusion of an 'F' natural as the final pitch. *No. 3* is similarly confined, although the FPF used is a centrosymmetric 7-STET collection {B, C, D, E♭, F, G♭} [1, 2, 1, 2, 1].

No. 6 is the first of my pieces to use a totally centrosymmetric and registrally fixed LIE scale throughout. The 76-STET scale (C♯1 to F7) consisting of minor third [3] and minor second [1] extensions from a Y1 E♭4 axis is laid out in bars 4–6 of the score (see Figure 26, below), after

[51] E.g. *With or Without Card* (Bang on a Can, 2019) and *Sixfivetwo* (Kronos Quartet, 2019).

[52] Having developed an interest in the "Black MIDI" concept (Connor, 2013), i.e. filling the scored page with as many notes as possible, I needed to find out the limitations of MIDI-to-cylinder construction, not least exactly how many notes could be crammed into any given temporal frame. Garnering this information proved problematic, and the piano rolls remain as yet unmade.

which ascending and descending sextuplets and semiquavers are rhythmically superimposed over and against each other, providing a fast contrapuntal landscape.

Figure 26: Music for player piano, No. 6 (bb.4–6)

At this early stage in my research, I was also considering the auditory aesthetic of George Antheil's *Ballet Mécanique* (1923–24) along with the Wurlitzer organ, but attempts to garner the appropriate information required to specifically compose for the latter were fruitless, and my initial “Black MIDI” sketches were temporarily abandoned. Similarly, my initial attempts to compose a chamber piece for clarinet and string trio based on (i) the registral limits of the ensemble and (ii) the triangular number 28 (intervallic structure = 7 + 6 + 5 + 4 + 3 + 2 + 1) were never realised satisfactorily. [53]

74 pitches 4 hands 1 piano (2017)
Piano Duet
Duration: 4’ 24”

Moving from 28 to (the next triangular number) 36 (+1, +2… +8), and from a *sounded* to a *silent* centre, the LIE scale for this piece was generated from E4/F4, the Y2 midpoint of a

[53] My “Black MIDI” concept is outlined here; sketches and scales derived from interpolated fifths and sequential conjunct tetrachords (triads and dyads) are explored. This work (although not completed) informs many of my subsequent pieces. The score, structural notes and MP3 audio file (an adaptation of a *live* workshop recording) for *28 Occasions of Experience* (2017) can be found in Appendix III of this thesis.

standard 87-STET (88 note) piano keyboard (A0-C8). The intervals are combined (nested) from the middle out, top down and bottom up, yielding the following 73-STET (26-pitch) scale from E1 to F7 shown in Figure 27. The intervallic pattern [2, 3, 2, 2, 5, 6, 5, 2, 2, 3, 2] is unique to the “concatenation of subsequences” (OEIS: A136436), an integer sequence that seems to apply specifically to the ↑↓ bidirectional combination of consecutive *counting* numbers.

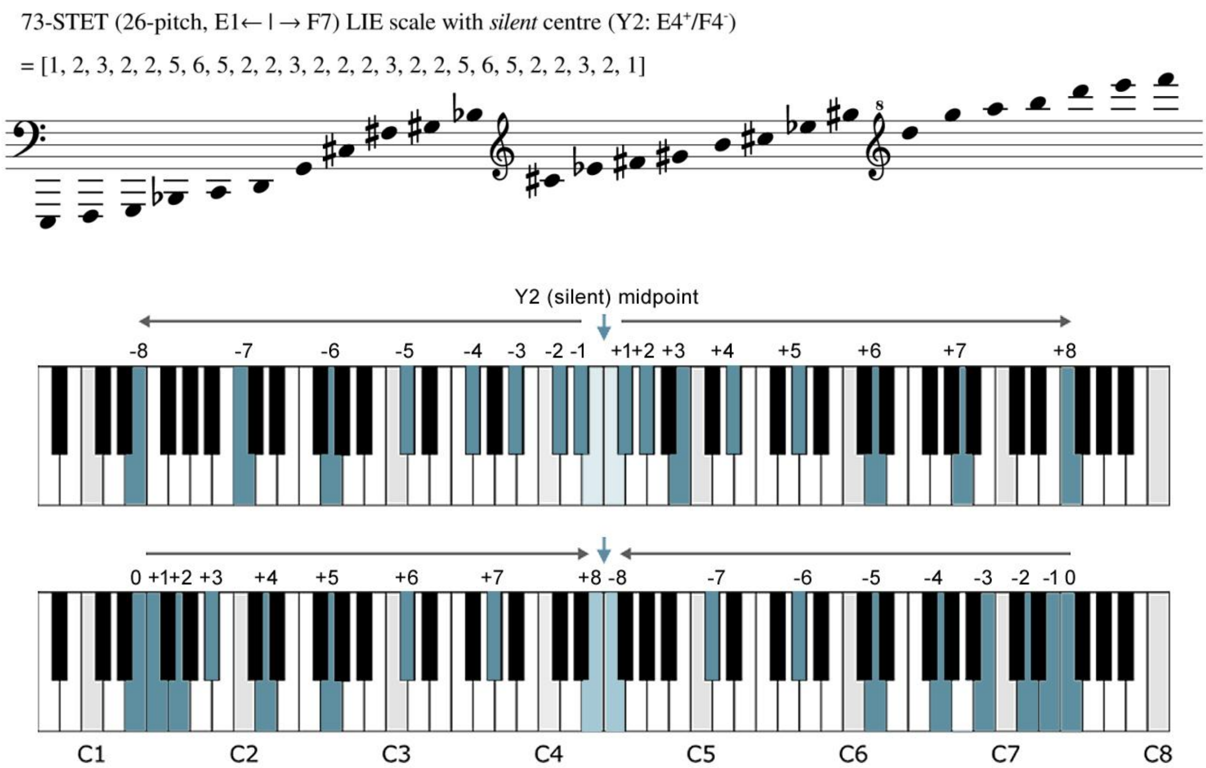


Figure 27: 73-STET scale

Composing this piece helped to initially formalise the use of the OG, an auditorily *relative* but architecturally *empty* or *alternative* space. As noted by Lichtenberg, “a good method of discovery is to imagine certain members of a system removed and then see how the rest would behave [Notebook J, 1571]" (cited in Wolfe, 2017, p. 139). The 69-STET (48-pitch, F♯1 to E♭7) *sounded* OG scale is as follows:

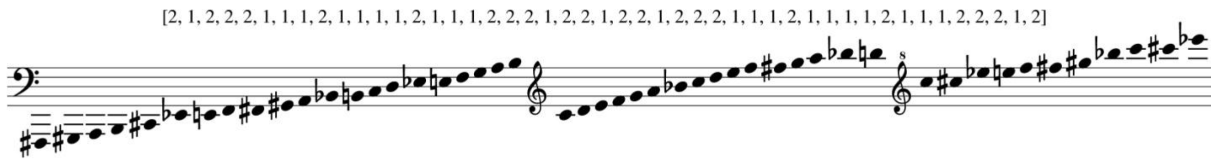


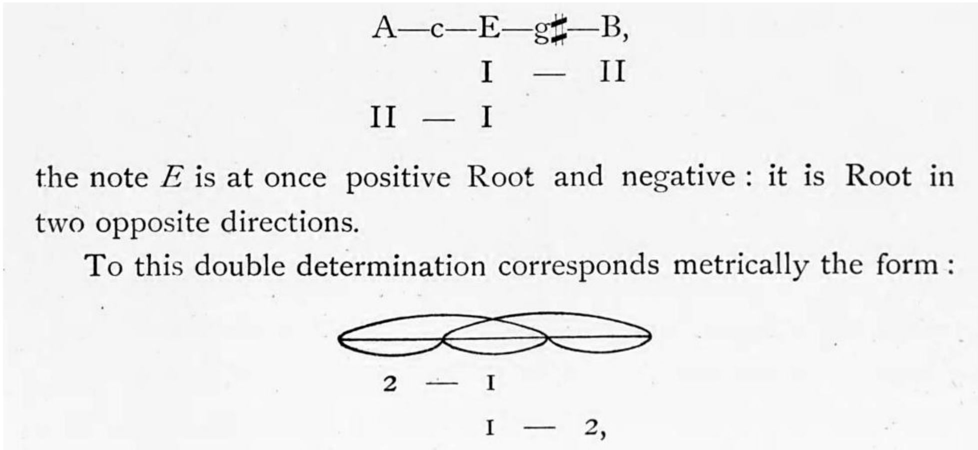
Figure 28: 69-STET OG scale

How these two sets are then deployed, juxtaposed, or superimposed, opens up diverse avenues of extended harmonic movement. This alludes to the "dipole" [54] (gravitational or magnetic) polarity potential of scalic combinations and explores the relevance of the vertices of a STET (range or span). Without piling on too many conjectures, AC/DC alternating current, polar attraction, and the cancelling effects of top spin and slice experienced in (e.g.) table tennis might be equally applicable analogies, and a more phenomenological means of approaching Ernst Levy's *absolute* conception of harmonic *telluric* polarity (1985).

Following on from Hauptmann *et al.*, Levy suggests that, with the minor triad, the "inner schism between structure and apperception is based on polarity" (Levy and Levarie, 1985, p. 15). According to Levy, although "we ought to hear the minor chord generated by C as C minor, ...*tellurically* we hear it as F minor" (ibid.). David Lewin credits Hauptmann's book *Die Nature der Harmonik und Metrik* (*The Nature of Harmony and Metre*, 1888) with introducing

[54] In organic chemistry, a dipole is described as a "*bond or molecule whose ends have opposite charges"* (Hardinger, 2017). What might constitute an auditory equivalent of the optical Faraday Effect or *Verdet constant* if all materials have "a unique Verdet constant that characterises its magnetic properties" (Foulkes *et al*., 2014, p. 392)? A physical example of bidirectional reciprocity is the seismic surface wave known as the Rayleigh Surface Wave. Whereas the particles in water move in a clockwise direction, in the case of Rayleigh surface waves*, "*particles at the surface trace out a counter-clockwise ellipse, while particles at a depth of more than 1/5th of a wavelength trace out clockwise ellipses. This motion is often referred to as being 'retrograde'" (Russell, 2016). An animated gif that best illustrates this motion can be seen here (ibid.).

into Western music theory the idea that “the philosophical principles underlying metric structure are the same as those underlying the harmonic structure of tonality” (Lewin 1981, p. 261). Hauptmann states that the “thoroughly natural and consistent metric determination …is in every respect to be identified with the harmonic determination… *It places in the middle the to-day, to which a yesterday and a to-morrow relate*” (1888, p. 227, emphasis added). Inverting and combining Hauptman’s {A, C, E, G♯, B} scale (see Figure 29) - ICs [3, 4 | 4, 3] + [4, 3 | 3, 4] produces {A, C, C♯, E, G, G♯, B} which retains its [3, 1, 3 | 3, 1, 3] 14-STET palindromicity (see Appendix II of this thesis).


A—c—E—g♯—B,
I — II
II — I

the note *E* is at once positive Root and negative: it is Root in two opposite directions.

To this double determination corresponds metrically the form:

2 — 1
1 — 2,

Figure 29: 14-STET LIE Scale; reproduced from Hauptmann, 1888, p. 225

Relating this sequence to a *named* OE scale, e.g. the Bygian scale (Zeitler, 2005) or Justin Pecot’s SOSian scale (2019) is perhaps pointless, as (and this is crucial to LIE scales as a 12-tone+ resource and concept), “inversional symmetry does not require equal octave division; relatively few inversionally symmetrical sets are also transpositionally symmetrical” (Morgan, 1998, p. 8). According to Ring (2016), there are only 52 reflectively symmetrical

12-tone scales that employ a major third as a maximum interval.[55] If a source scale with quasi-12-TET tonality is required, then further extensions to the above scale in both directions (e.g.) ICs [1, 3, 1, 3, 1, 3 | 3, 1, 3, 1, 3, 1] produce a two octave 24-STET repeat/loop – {E, F, G♯, A, C, C♯, E, G, G♯, B, C, E♭, E} the PCs of which correspond to the nonatonic *Genus Chromaticum* scale (AKA Messian's Mode 3 and Tcherepnin's Augmented Ninth); the corresponding OG scale would be D, F♯, and G♯. If, as Lewin suggests, Hauptmann's treatise is indeed the entry point for a philosophical and theoretical conjunction of metric and harmonic elements, Hauptmann's extensive use of reciprocal centrosymmetric structures [56] (if nothing else) is a justification for further enquiry.

In *74 pitches 4 hands 1 piano,* at b. 21 (one bar before Letter B) the piece switches from prime to OG scalic material, then prime elements are gradually reintroduced before the OG exerts its dominance through a series of common time ascending arpeggios (Letter D, b. 49) followed by a reiteration of the 13/16 *walking* bass line with accompanying chordal clusters. At Letter H (b. 81) the prime scale begins to finally gain control of the upper then lower registral ranges, culminating in the *fff* chord at bar 84. However, as large swathes of this piece have favoured the OG collection, this scale has the last laugh (or death rattle) through the reiteration of the opening thematic demisemiquaver pattern (from bb. 4–20) transposed into OG pitch material at bar 82. The two fingered staccato techniques (first introduced at Letter B, b. 22) were employed as a (naive and/or percussive) means of forcing the timing of temporal accents. At the time of composing I was not consciously aware of Lionel Hampton's

[55] See Ring's section on "reflective symmetry" (2016).

[56] See p. 229 regarding "organic impossibility", pp. 245–250 for Gothic arch structure, and p. 274 for "Hexameter + pentameter = 11". Also of interest is Hauptmann's "Analogy in the Determination of Space…" (p. 288). See the Appendix (p. 348) for an explanation of Hegelian metaphysics and a short analysis of the whole book. See Riley (2004) for the oppositional contrast between Heinrich Schenker's (overtone reliant, *natural* major and *artificial* minor) major-minor system and Hauptman's "third mode, the 'minor-major' (*Moll-Dur*)" (Riley, 2004, pp. 2–4).

two-fingered piano technique (derived from his ambidextrous vibraphone sticking), as used in the tracks *Space Man* (1938) and *Two Finger Boogie* (1945),[57] but listening to and comparing Hampton's technique to this piece serves to highlight the uniqueness of a LIE scalic sound world. In conversation, Julien Kottukapally (pianist) said that he "very much enjoyed learning this piece".[58] My next step was to explore scales that specifically utilised (and were generated using the principle of) scalic directionality.

A gilded cage (2017)
Scored for solo flute
Duration: 5' 00"

A gilded cage began as an exercise to compose a five-minute piece for a solo instrument, as suggested by Peter Wiegold. This five-minute challenge warranted the pre-compositional consideration of a metrical centrosymmetric structure, and although I am not an advocate of strict *integral* or *total* serialism per se, I often return to and try out (pitch/pulse) integral processes during preparatory compositional procedures. Having listened to various different flute techniques and deciding to juxtapose the (*da*) slap tongue with flutter tongue, the distribution of these elements along with a dynamic curve were all integrated into the large-scale scheme (see Figure 30, below).

[57] Hampton's two fingered technique can best be seen in this footage from a 1978 live concert in Holland (Thejazzsingers channel, 2017).
[58] *74 pitches…* was initially composed with the Brunel University Associate Ensemble *Piano Circus* in mind.

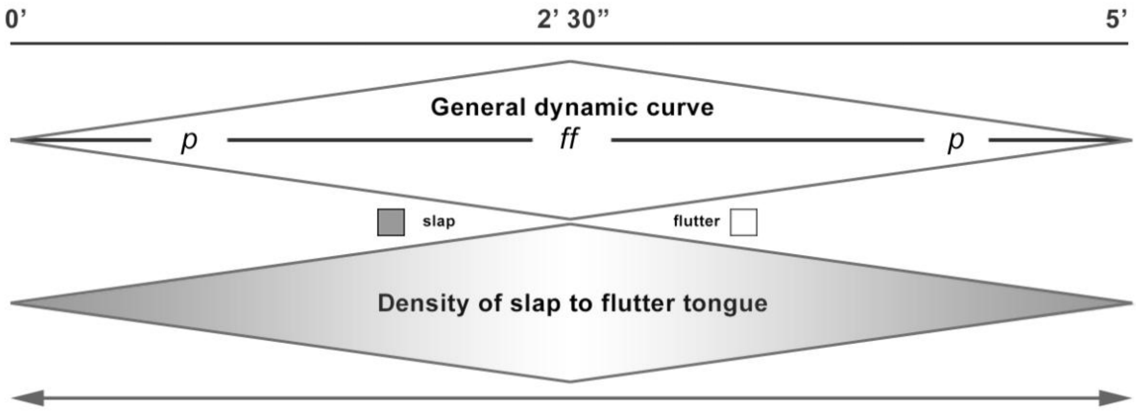


Figure 30: Density and dynamic curves; *A gilded cage* (2017)

The LIE scale used is derived from the Y2 midpoints (G♯5/A5) of the flute range (C4 to F7), again using a run of consecutive counting numbers [6, 5, 4, 3, 2, 1, 2, 3, 4, 5, 6]. This piece was inspired by observing the (seemingly random) up and down motion of midges, [59] and the ascending *arohanam* and descending *avarohanam* sequences of Melakarta ragas (Balasubramanian, 2002, p. 2); as this research progressed, the use of Melakarta ragas became increasingly important. Theoretically, in-between rests (and within the ornamentation of each phrase and/or crotchet) the Prime LIE scale is favoured when ascending, and the corresponding OG when descending (see Figures 31 and 32).

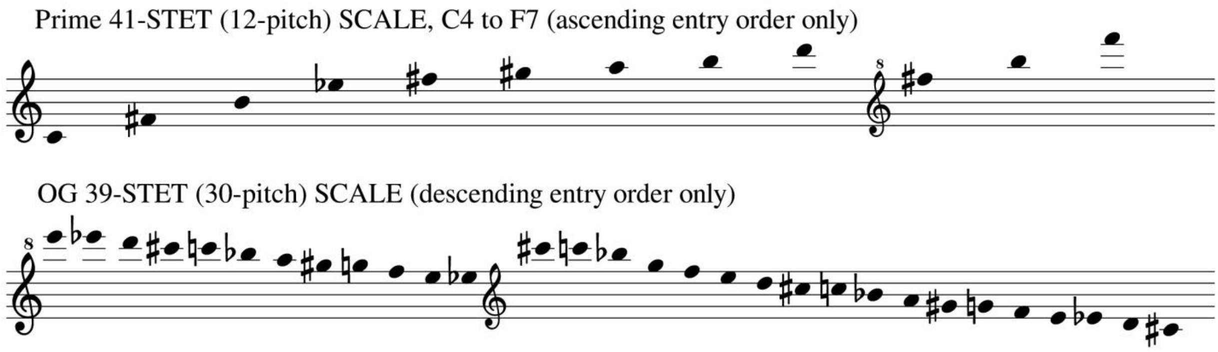


Figure 31: Prime and OG scales; *A gilded cage* (2017)

[59] If one focuses on a midge cloud as one might on a flat holographic image, i.e. looking *through* rather than *at* the image, a certain number of midges can appear to rise and fall in unison.

For example, the directional attributes of bar 25 are as follows:

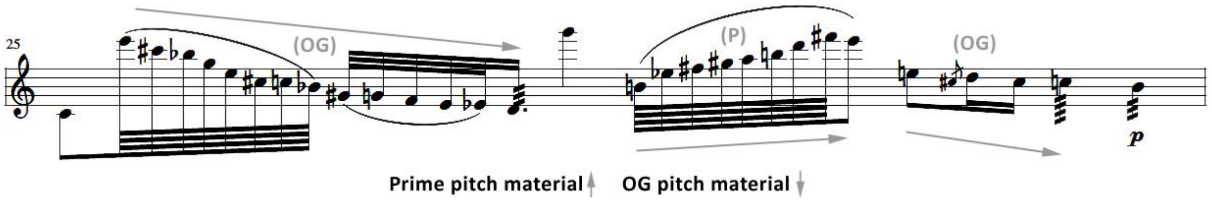


Figure 32: Scalic direction; *A gilded cage* (2017)

The extent to which these directional constraints are audible or meaningful is debatable. Might the directionality of stretched constructs engender an alternative mode of (possibly holistic) listening, beyond pitch or interval class commonality, Morris's "similarity index" (1979, p. 446) derived from "interval-class-vectors" (ibid.) and/or Lewin's *canonical equivalence* based on an "embedding number EMB (X, Y)... defined as the number of forms of X that are included in Y... a kind of a 'transitivity formula'" (Vuza, 1988, p. 264)? What writing in this manner does provide is a hands-on insight into both monophonic counterpoint and puzzle cannons, while also highlighting the daunting possibility that each LIE scale might require its own form of extended dissonant counterpoint (or not). At this juncture, I had not yet discovered Hugo Kauder's use of specifically *directional* combinatorial scales.

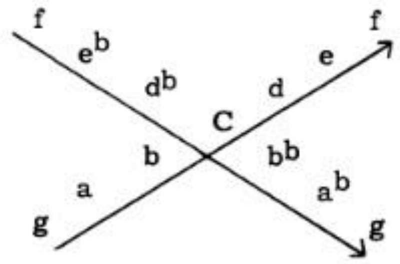


Figure 33: Kauder's "double scale", based on Ritchie, 1961, p. 139

According to Kauder, the scale pictured above (Figure 33) is the "combining of two opposite basic scales on one tone" (Kauder, 1960, p. 9) which produces "one hermaphroditic double scale of eleven tones" (ibid.). The tones of the F to G scale occur only in descending order, while the G to F scale is applied in ascending order. Kauder refers to the major (Ionian) scale as "the Rising Scale after the rising frequencies of the harmonic series, the Phrygian mode the Falling Scale after the inversion of the harmonic series, and the Dorian mode the Neutral Scale" (cited in Goldblatt, 2012, p. 19). Kauder's 10-STET (eleven-tone) double scale is essentially the chromatic minus the tritone (F♯) from G ↑ F, and what Kauder calls "the most perfect form of the pentatonic scale" (1960, p. 16) is the 10-STET centrosymmetric construct {A, C, D, E, G} or [3, 2, 2, 3]. Kauder does not appear to overtly take registral concerns into consideration, without which, the Dorian may perhaps be perceived as being *more* neutral.

In conversation, flautist Amelie Donovan found the passages that combine rapid grace notes with flutter (e.g., b. 5) to be particularly "tongue twisting". However, Amelie spent many days learning, performing (in a class and end-of-term recital at the Royal Welsh College of Music & Drama, RWCMD) and thoroughly engaging with this piece, and the result speaks for itself. [60] Having pursued the idea of intervallic *directionality*, I next explored the timing of onset events.

[60] Regarding the temporal feasibility of performance, Ian Anderson's flute solo (listen here, 1':31") on Track 8 of Jethro Tull's 1978 live album "Bursting Out" (Jethro Tull, 2017) was both a reference and an influence. Listening back to this piece at twilight, I am tempted to suggest that it is best performed and heard in a remote wood during the twilight chorus. Adding a field recording of said chorus to the sound file was also tempting but seemed too literal and crass. *As birds fly backwards* (2017) uses the same ascending only 'A' (LIE) scale juxtaposed with a descending only 'B' (OG) scale, along with similar cellular (pitched and rhythmic) material. The score, sound file and structural concerns for this piece can be found in Appendix III.

Outographic No.1 (2018)
Scored for Cl., Bsn., Trp., Trb., Vln., Db., and Perc. (Glk. and Xyl.)
Duration: 4’ 20”

Originally composed for an *Ensemble Mise-En* (NY) “call for scores”, the generative LIE scale behind this piece is the following 89-STET (G0 < > C8) scale.

Figure 34: LIE scale; *Outographic No.1* (2018)

However, as implied by the title, it is the 85-STET (A0 < > B♭7) 64-pitch OG field that is here used. This particular scale has some unique sequential attributes, not least a correspondence with the “Leonardo logarithm of n” (OEIS: A001179), the “prime signatures… of the positive integers” (OEIS: A118914), and p-adic (3-adic) numbers (OEIS: A280509), as follows:

Figure 35: Attributes of the OG scale; *Outographic No.1* (2018)

After listening to Stravinsky’s *Symphonies of Wind Instruments* (1920), I wondered what a similar timbral world might sound like without the overt use of “tutti”. Similarly, might (e.g.) the unforgiving *ostinati* in Jean Françaix’s *Wind Quintet No.2* (1987) be gradually (or more implicitly) approached with rhythmical stealth and numerical balance, thus exploring the extent to which “beat perception is possible when attention is not directed at a rhythm” (Bouwer *et al*., 2016, p. 80)? [61] The subliminal numerical engine that underpins the onset entry order is based on the (↑↓) symmetry of interval, combined with a silent durational value of seven crotchets.

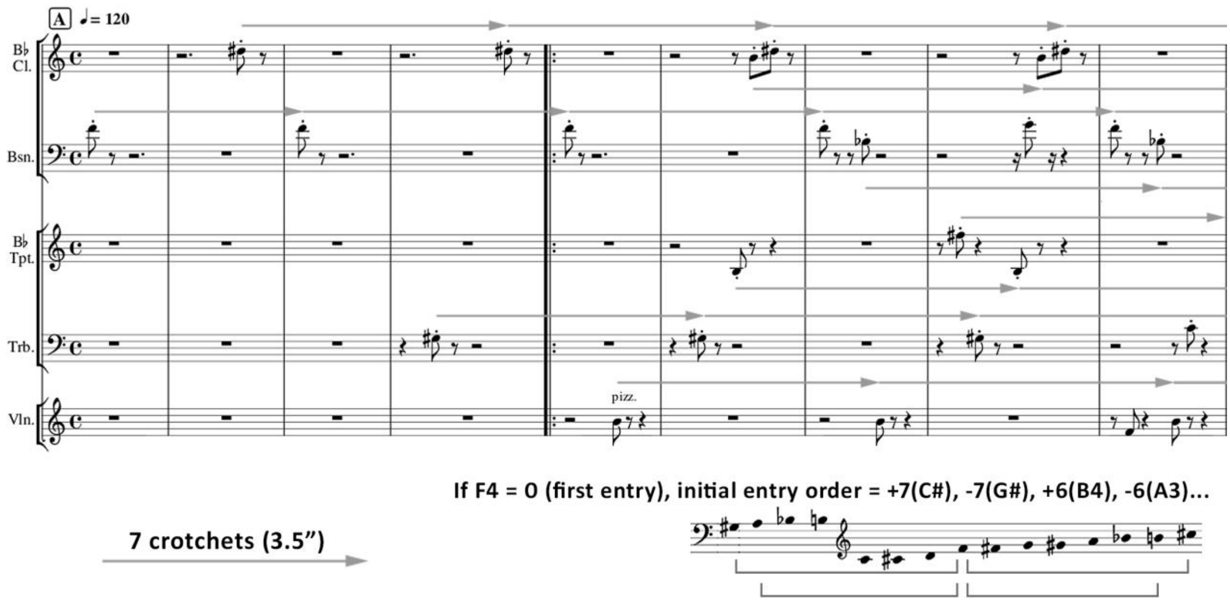


Figure 36: Temporal onset entry order; *Outographic No.1* (2018)

This reductive numerical entry order device drastically simplifies the rhythmic complexity of the individual parts. When all parts are sounded, a crotchet (1/4) pulse is evident by bar 4, a

[61] Within my overarching desire to articulate a relationship between the OG emptiness of pitch space and the relative emptiness of temporal intervals, the Japanese aesthetic of *Wabi-Sabi* was also useful, *wabi* being (broadly) a “philosophy of spatiality, direction or path” (Sverdrup, 2013), with *sabi* relating to “an aesthetic of objects and their possession of time” (ibid.). Sverdrup uses the phrase “an expectant stillness that holds the possibility of change” (ibid.), which perhaps best sums up my compositional intent for this particular piece.

quaver (1/8) pulse is evident from bar 6, and a semiquaver (1/16) pulse is first alluded to with the bassoon entry in bar 8. Although this implicit (1/16) substrate pulse is arguably *felt* from bar 22 onwards, it is never explicitly delivered or consciously heard for longer than five consecutive crotchets (bars 25 and 26) and then only for one bar at b. 63. The coherence of this underlying metrical framework is of course dependent on strict tempi adherence; the better the adherence, the better the outcome. The first glock entry (letter C, b. 30) reinforces then immediately thwarts expectations, which also masks the fact that the silence between onsets has now been stretched from seven to fifteen crotchets (approx., see diagram below).

Figure 37: Bars 30–38; *Outographic No.1* (2018)

The limited palette of staccato pitches, combined with the staggered onset entries, gives focal prominence to the timbre of each individual instrument, and therefore also explores what is known as auditory "gist", the "recognition of very short sounds from timbre cues" (Suied *et al.*, 2014). The staccato entries seem to expand and contract (in keeping with the scale itself), essentially "folding" and "unfolding" temporality with the aid of timbral juxtaposition, an emerging klangfarbenmelodie (of sorts). The *pointillist*/*punctualist* [62] idea expressed in this piece is something I return to in many of my larger-scale pieces.

Examples of compositions with punctualist elements include Webern's *Quartet*, Op. 22 (1928–1930) and *Concerto for Nine Instruments*, Op. 24 (1934), Babbitt's *Three Compositions for Piano* (1947), Goeyvaerts's *Sonata for Two Pianos* (1951), Boulez's *Structures I* (1952), Hans Werner Henze's *Prison Song* (1971), Crumb's The *Magic Circle of Infinity* (*Makrokosmos Volume 1*) (1972), Penderecki's *Threnody* (bb.26–63) (1961) and *Quartet for Clarinet and String Trio*, Mvt. III (1993).

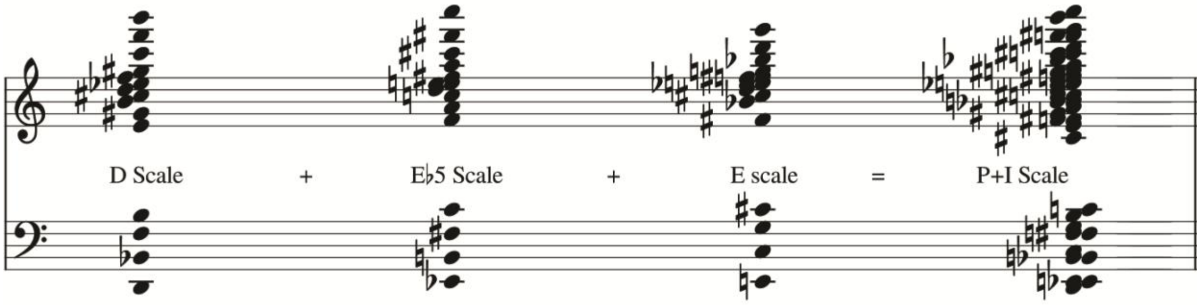
Outographic No.1 is simply a LIE scalic based exploration of punctual/temporal auditory art; around the same time (2018) I also composed *Heisenberg on Bourbon Street* and *Even (odd) Scale* (see scores in Appendix III). [63]

Notes of Protest (2019)
Duet for Cello and B♭ Clarinet (virtual or live)
Duration: 2′ 12″

Prior to composing this piece, I worked on a duet for Xylophone and Piano (*3′ 44″*) and a collection of *13 piano études* (see Appendix III) that, among other things, explore (i) the 38-STET sequence [1, 1, 1, 1, 5, 1, 7, 2, 3, 5, 11] (previously mentioned in relation to octaves of resolution in Part 1 of this thesis), and (ii) what I have called a symmetrical "rubato rhythm", which is further developed in *Symphony No.2* (2022).

[62] In the first of his 1972 lectures, Stockhausen refers to this as *point* and/or *star* music (Olano, 2013).
[63] *Even (odd) Scale* (2018) is derived from a randomly generated scale and *Heisenberg…* (2018) explores a randomly generated scale combined with a consecutive counting number (+1, +2… +7) variant.

During lockdown (2020) I worked with the American cellist Michael G. Ronstadt on a virtual recording project for Shelter Cymru,[64] and later persuaded Michael to record this short piece to a click track. As written on the cover page of the score, the performance instructions are to imagine that "after decades underground, a post-apocalyptic cellist emerges into daylight (having only ever heard one broken Charlie Mingus record) and attempts to jam with the solitary bird that's made it out for this new dawn chorus" (Bizzell-Browning, 2019). Michael spent a few days learning and working on this piece, and aside from various discussions regarding changing around a couple of the shorter arco and pizzicato sections, the only scored alteration was to slow the tempo from the original 128 BPM to 118 BPM; it seems that I was asking too much in this department, but the outcome was to our mutual satisfaction.



D5 scale {D2, B♭2, F3, B3, E4, G♯4, B4, C♯5, (D5), E♭5, F5, G♯5, C6, F6, B6} [8, 7, 6, 5, 4, 3, 2, 1, 1, 2, 3, 4, 5, 6]
E♭5 scale {E♭2, B2, F♯3, C4, F4, A4, C5, D5, (E♭5), E5, F♯5, A5, C♯6, F♯6, C7} [8, 7, 6, 5, 4, 3, 2, 1, 1, 2, 3, 4, 5, 6]
E scale {E2, C3, G3, C♯4, F♯4, B♭4, C♯5, E♭5, (E5), F5, G5, B♭5, D6, G6} [8, 7, 6, 5, 4, 3, 2, 1, 1, 2, 3, 4, 5]

D5 + E♭5 + E = P+I (combinatorial) scale:
{D2, Eb2, E2, Bb2, B2, C3, F3, F#3, G3, B3, C4, C#4, E4, F4, F#4, G#4, A4, Bb4, B4, C5, C#5, D5, Eb5, E5, F5, F#5, G5, G#5, A5, Bb5, C6, C#6, D6, F6, F#6, G6, B6, C7}

Utilised Clarinet pitches: {F♯3,G3,B3, F4, F♯4, A4, B♭4, C5, C♯5, D5, E♭5, E5, F5, F♯5, G5, G♯5, B♭5, C6, C♯6}
Utilised Cello pitches: {D2, E♭2, E2, B♭2, B2, C3, F3, F♯3, G3, B3, C4, C♯4, E4, F4, G♯4, A4, B4, D5, E♭5}

Figure 38: Source scales used in *Notes of Protest* (2019)

[64] As a busker I had previously (on occasions) lived on the streets, so when the pandemic struck, my thoughts were with the homeless. In partnership with Shelter Cymru (the Welsh people and homes charity) I produced the album Gwaith Cartref (2020) which raised a few thousand pounds for charity, and involved over 30 cross-genre musicians from around the world recording virtually. *Gwaith Cartref* (which means *homework* in Welsh) by the Lockdown Collective can be heard here. This work was inspired by the surrealist parlour game known as "consequences", i.e. small musical phrases, rhythms and riffs were passed around between the musicians resulting in the 10 unique tracks that make up the album. Many of the initial ideas emanated from Wales based guitarist and teacher Mark Ellwood.

The combined ranges of the Cello (C2 - C6) and B♭ clarinet (E3 - C7) are C2 - C7 (approx.), and in this instance I chose to place the central LIE scalic axis at E♭5. *Notes of Protest* utilises the combination of three basic (…3+2+1+0+1+2+3...) counting number runs that are extended, within range, from three adjacent semitonal starting (zero point) pitches: D5, E♭5, and E5 (see diagram above). The result of this is that the complete chromatic is sounded over a 14-STET central span, which extends from G♯4 to B♭5, a perfect fifth either side of the E♭5 axis. All the pitches of the P+I (combinatorial) scale from D2 to C♯6 are sounded at least once. Strictly speaking, the clarinet trills (B♭5 and C♭6) in bars 3 and 8 should be sounded as A5 and B♭5, and the cello C2 semibreve (bar 14) should be scored as D2, but I grew accustomed to these scored mistakes. This piece is a product of its time, a time when *nothing* was performed in front of a live audience and *all* performances were necessarily virtual. Although this piece might now (potentially) be performed by a live duet, retaining the option for a virtual (pre-recorded or midi-enabled) clarinet part is evocative of this unprecedented historical moment.

3313133 (2021)
Scored for Vibraphone, Marimba, and Piano
Part I – *Duration: 2' 35"*, Part II (Solo Piano) – *Duration: 2' 54"*, Part III – *Duration: 1' 46"*

Carl Faia suggested I listen to Bartók's *Mikrokosmos* (1926–39), and *Divided Arpeggios* [65] from Book VI was of particular interest regarding architectonic structure and the progression of "cells" (pitch collections) from various central "axial" points over various spans.

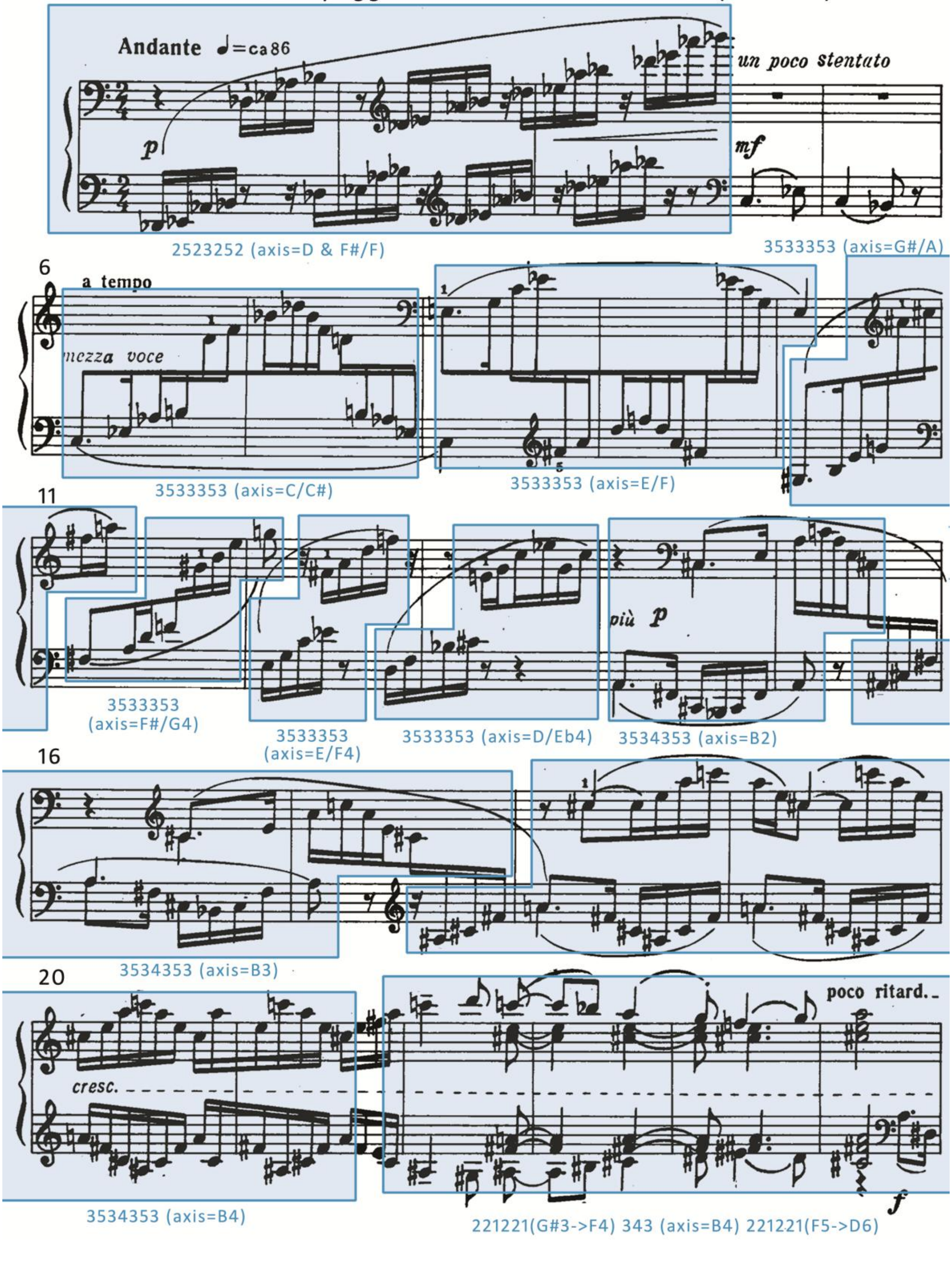
[65] Score-Video with streaming sound on YouTube (Breemer, 2020).

Robert Katz uses the term “axis of axes” (1993, p. 336) to describe the principle where “a primary axis (or possibly a pair of axes) forms the vertex of a symmetrically balanced system” (1993, p. 336). With regard to Yavorsky’s single and double “identity”, there are of course 24 possible axis points within any 12-tone set (C, C/C♯, C♯, C♯/D, D... B♭, B♭/B, B, B/C) and potentially many more within any registrally extended scale.

From a symmetrical analysis of *Divided Arpeggios* (see following pages), the intervallic cell [**3, 3, 1, 3, 1, 3, 3**] that Bartók deploys at bb. 31–35 is explored as a compositional building block. This symmetrical cell is here used as both an extended intervallic pattern (a 17-STET scale) and a 17 beat rhythmical framework.

Symmetrical axial analysis of

Bartok's *Divided Arpeggios - Mikrokosmos Book VI* (1926-39)




26
55 (axis=A4)
accel. al tempo
3433343 (axis=B3)
espr.
11 (axis=F#4)
dim.
p
f
3313133 (G3->C5, axis=Eb/E4)
32
cresc.
dim.
37
sotto
p
sopra
poco ritard.
42
cresc.
f
sopra
sotto
a tempo
353(3)353 (axis=B4/C5)
353(3)353 (axis=G/G#4)
48
mf

Figure 39: Symmetrical analysis of *Divided Arpeggios*

The central point within the sounding pitch range of a four Octave Marimba and Vibraphone (C3-C7) is C5. If we approximate the *double* Y2 centre of this extended cell to be B4/C5 and treat both of these pitches as a silent axial point, the minor third interval from B♭4 to C♯5 corresponds to the central interval of the [3, 3, 1, **3,** 1, 3, 3] scale. By extending this sequence from the centre outwards, within range, the following 45-STET (C♯3 to B♭6) pitches are produced. This scale has three silent Y2 axial points at F♯3/G3 - B4/C5 - E6/F6.

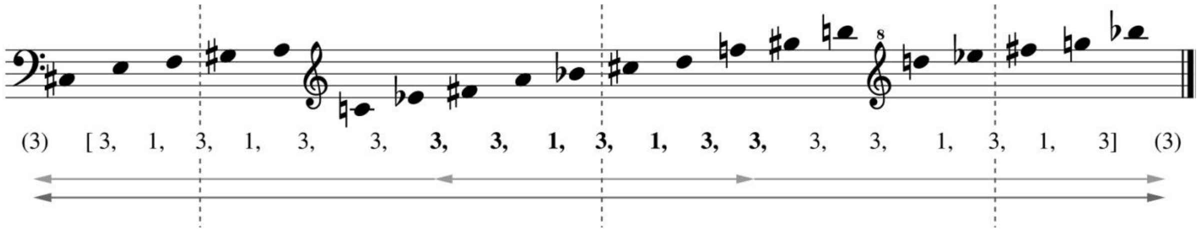


Figure 40: 45-STET scale; *3313133* (2021)

In order to fully express a strict mathematical (but beyond range) threefold (17 x 3 = 51-STET) version of the above scale, the pitches B♭2 and C♯6 would need to be added onto either end. By doing this, the PC cardinality (a guide to the internal OE intervallic construction of any centrosymmetric scale) of this particular 22-pitch FPF is as follows (see diagram below).

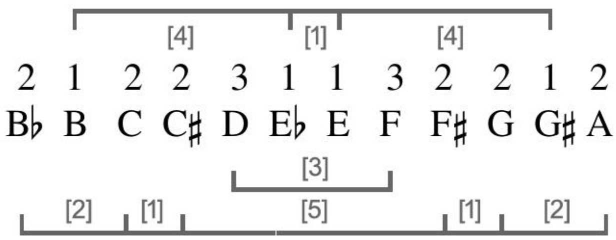


Figure 41: PC cardinality; *3313133* (2021)

Arguably the least used triads within this configuration are [4, 1] and [1, 4], with [4, 1, 4] being the least prevalent tetrachord; these intervals (along with the inverse [1, 4, 1] tetrachord) were subsequently explored in my next two pieces *[1, 4] cells* and *Vincent and the Maverick Sonority* (see Appendix III of this thesis).

Adding a piano to the instrumentation of this piece obviously extends both the timbral and scalic possibilities. Within the piano range, the above 45(and/or 51)-STET scale can be extended to 86-STET (37-pitches, see diagram below).

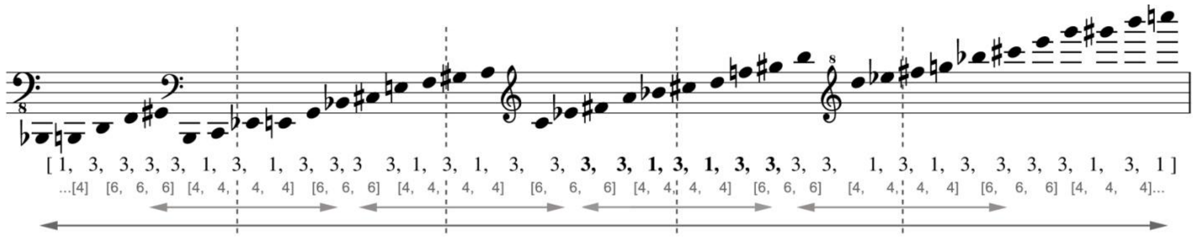


Figure 42: 86-STET scale; *3313133* (2021)

This scale has four silent Y2 axial points at C♯2/D2 [17] F♯3/G3 [17] B4/C5 [17] E6/F6 (five axial points if the next available +17, although truncated, A7/B♭7 axis is included), and in OE terms suggests a conjunct stacked ladder of major thirds [4] and tritones [6]. Whereas the cycle of fourths [5] neatly maps out the complete twelve pitches of the chromatic over a 60-STET (12 x 5) span, extensions of a major third never progress beyond the augmented triad, and tritonal extensions beyond a dyad. To abstractly (beyond sounding range) map out the total chromatic using 17-STET extensions would require a 204-STET (12 X 17) span (e.g. C [17] F [17] B♭ [17] E♭ [17] G♯ [17] C♯ [17] F♯ [17] B [17] E [17] A [17] D [17] G [17] C). It would be banal to suggest that simply through magnitude an extended FPF variant alludes to infinity more than a perceptually (perhaps more) static octaviated variant, but by examining the PC cardinality of 17, 51 (17 X 3), 85 (17 x 5) and 119-STET (17 x 7) scalic extensions from a B/C Y2 axial centre, it becomes evident that the intervallic flavour of these collections is not what might be expected within 12-tone PC sets (see Figure 43, below). As the scale expands, the predominant PCs fluctuate around the B/C axis, and the prevalence of the axial B/C pitches increases proportionally in relation to odd number multiples of 17. I.e. if n is an odd number, the prevalence of axial pitches = STET length ÷ n – 2. Although this seems to only apply to the axial pitches in this *eleventh* (or *compound* [66] fourth) scenario, perhaps these particular LIE scales provide alternative insights into the perception of infinite progressions from finite structures.

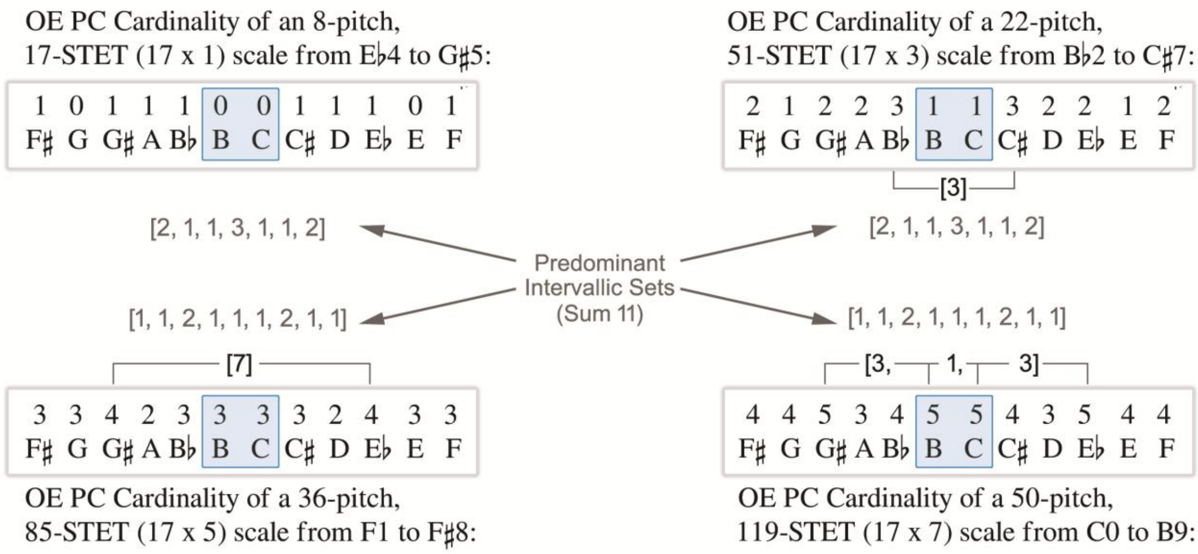


Figure 43: PC cardinality, STET multiples of 17

Whilst exploring the auditory and rhythmical possibilities of the above scale(s), I tried mapping the 37-pitches of the 86-STET (prime) scale (see Figure 42, above) onto various existing pieces from the art-music canon. For example, using Chopin's *Scherzo No.1 in B minor* Op. 20 (1835), the rhythm remained the same, but the pitches were shifted to their nearest counterpart within the 86-STET scale. This experiment suggested that over relatively short time frames, Chopin's rhythmicity was as intrinsic to his "sound" as any specific harmonic movement.[67]

[66] Why (in musical discourse) is the term *compound* ascribed to intervals that are larger than an octave, yet a compound *chord* tends to refer to chordal combinations regardless of octaviated distance, and compound *time* refers specifically to triplets?
[67] A brief MP3 example of this prime scale repurposing or mapping can be heard here.

Figure 44: 17 consecutive semiquavers; Chopin's Scherzo No.1 in B minor Op. 20 (1835)

Within the 3/4 pulse of Op. 20, in the right hand, Chopin uses 17 consecutive semiquaver onsets throughout (as annotated in Figure 44, above), often 8 bars apart (e.g. b. 5, b. 13, b. 125, b. 133, b. 249, b. 389, b. 397 and b. 513). Due to the minimal number of divisors related to 17, there are relatively few symmetric intervallic patterns that can be derived from extensions of the [3, 3, 1, 3, 1, 3, 3] structure. One pattern of interest is a combination of tritones and fifths [6, 5, 6 + 6, 5, 6], which, ↑↓ from E♭4 produces a subset 68-STET (17 x 4) FPF from F1 to C♯7, with four partitions: F1 [17] B♭2, B♭2 [17] E♭4, E♭4 [17] G♯5, G♯5 [17] C♯7.

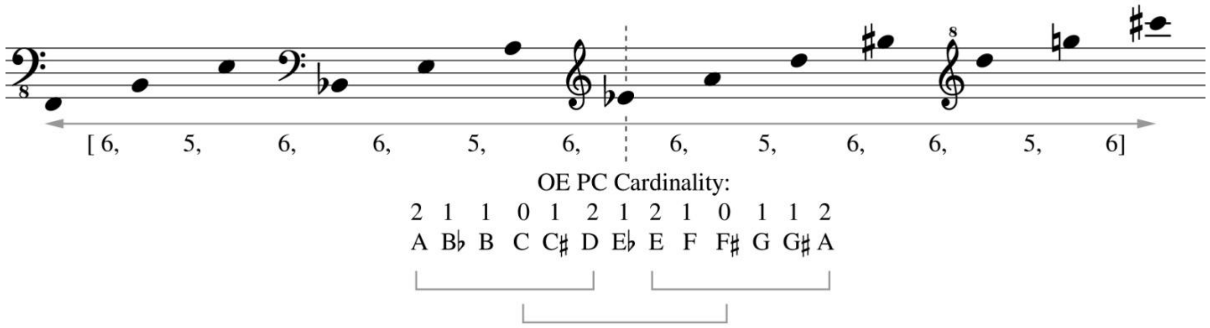


Figure 45: 68-STET scale (F1 to C♯7) with four partitions

The PC cardinality of this scale suggests the prominence of two adjunct fourths or conjunct fifths [5, 2, 5] and omits the tritonal (C and F♯) pitches either side of the E♭ axis; intriguingly, in the construction of *Structural Declaration No.1* (discussed next in this section), the concept of scale step (sum) construction involving ↑5, ↓2, ↑5 movement results in the 17-STET hexachords [8, 3, 6] and [6, 3, 8].

Various combinations of, and juxtapositions between, the above mentioned scales and 17-beat durations are explored throughout the three parts of *3313133*, from the opening scalic unfolding of a centrosymmetric 34 (17 x 2) quaver pattern in the marimba part (Part I), that unfolds the dyads of the prime scale (B♭4/C♯5 ↑↓ B♭6/C♯3, see Figure 46, below), to the vibraphone part at the end of Part III (bb. 13–25) that repeats this idea over a 17/8 time signature with different interval onsets; the three differing tempi markings for the three sections are a compromise between aesthetic and practical performance considerations.

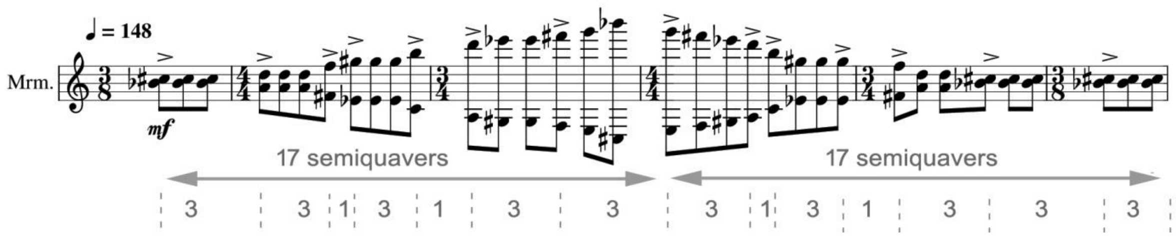


Figure 46: Marimba part (Part I); *3313133* (2021)

The solo piano section (Part II) sits in the middle, thus forming a large scale timbre-related ABA (arch) structure; as used by Penderecki to symmetrically displace the percussive and punctualist elements in *Threnody* (1961) (see Kozak, 2016, p. 208) and (inversely in) *Polymorphia* (1961). Along with various canonical devices inspired by the Shepard scale, Part II also explores the placement of dynamic accents over a quaver pulse (example below), and this more overtly "jazz" inspired (4 + 3 + 4 + 3 + 3) section might be thought of as *Take 17,* a rhythmical extension of Dave Brubeck's *Take Five* (1959). Part III combines elements from Parts I and II.

Figure 47: Part II, piano part; *3313133* (2021)

Letter D to F of Part I moves to and from the 85-STET OG scale and the 86-STET (B♭0 to C8) Prime (P+I LIE) scale.

Figure 48: 85-STET OG scale; 3313133 (2021)

Structural Declaration No.1 (a piano concerto) (2021)
Scored for large ensemble and piano
Duration: 9’ 00”

ALL-INTERVAL 12-NOTE CHORDS
(SYMMETRICAL INVERTED)[8]

1: 274316E985T	23: 523416E89T7	45: 45E926T3178	67: 473E26T1958
2: 134726T589E	24: 1432567T98E	46: 29E5468713T	68: 2E37468591T
3: 274916E385T	25: 3E8726T5419	47: 45E326T9178	69: 4ET73695218
4: 194726T583E	26: 278E369145T	48: 23E5468719T	70: 37TE4681259
5: 4523E619T78	27: 3145T6278E9	49: 497126TE538	71: 4ET79635218
6: 19T7864523E	28: 278E963145T	50: 217946835ET	72: 3521864ET79
7: 4529E613T78	29: 491726T5E38	51: 437126TE598	73: 5TE83694127
8: 13T7864529E	30: 27194683E5T	52: 217346895ET	74: 38ET5672149
9: 589T16E2347	31: 431726T5E98	53: 358126TE479	75: 5TE89634127
10: 1T98567432E	32: 27134689E5T	54: 218536974ET	76: 3412765TE89
11: 583T16E2947	33: 4137T6259E8	55: 374ET621859	77: 54E2369T187
12: 1T38567492E	34: 259E864137T	56: 218596374ET	78: 32E456781T9
13: 347216ET589	35: 4197T6253E8	57: 538ET621497	79: 54E2963T187
14: 127436985TE	36: 253E864197T	58: 214976538ET	80: 3T187654E29
15: 385TE612749	37: 413526T79E8	59: 598ET621437	81: 45T1369E278
16: 127496385TE	38: 2531468E97T	60: 214376598ET	82: 31T546872E9
17: 3T7816E4529	39: 419526T73E8	61: 4791T62E358	83: 45T1963E278
18: 187T369254E	40: 2591468E37T	62: 2E35864791T	84: 3E278645T19
19: 3254E6187T9	41: 4E5326T9718	63: 4731T62E958	85: 592E4681T37
20: 187T963254E	42: 235E468179T	64: 2E95864731T	86: 4E295673T18
21: 529416E83T7	43: 4E5926T3718	65: 479E26T1358	87: 4E235679T18
22: 1492567T38E	44: 295E468173T	66: 2E97468531T	88: 532E4681T97

[8] The letters “T” and “E” represent the intervals of ten semitones and eleven semitones respectively.

Figure 49: All-Interval 12-Note Chords (Carter, 2002, p. 57)

The LIE scalic FPF scaffolding of this piece is derived from combining two of Elliott Carter's symmetrically inverted “All Interval Twelve-Note Chords” (reproduced above). Originally compiled by Carter in 1979, these chords (scales or fields) are notated using “inversionally related intervals... distributed in opposing directions around a tritone” (Carter, 2002, p. 54). There are 88 of them, all of which (as mentioned by both Babbitt and Slonimsky) have an intervallic sum of 66, here referred to as a 66-STET span. Carter’s 88 chords are half the number of the 176 all-interval series (AIS) with “tritone nests” identified by Morris and Starr (1974, p. 372), which they had in turn whittled down from a possible 3,856 “transpositionally and rotationally normal series” (ibid, p. 366). From Carter’s list, for example, chords 2 and 4 only differ due to their major sixth [9] and minor third [3] inversions (1**3**4726T58**9**E -

1**9**4726T58**3**E). When P+I superimposed and combined (neither of which Carter specifically alludes to) [68] these two symmetrically balanced collections produce a 66-STET (33-pitch) set, here used as a Prime scale, using an axis of symmetry at C4 (i.e. E♭1↔C4↔A6), as follows:

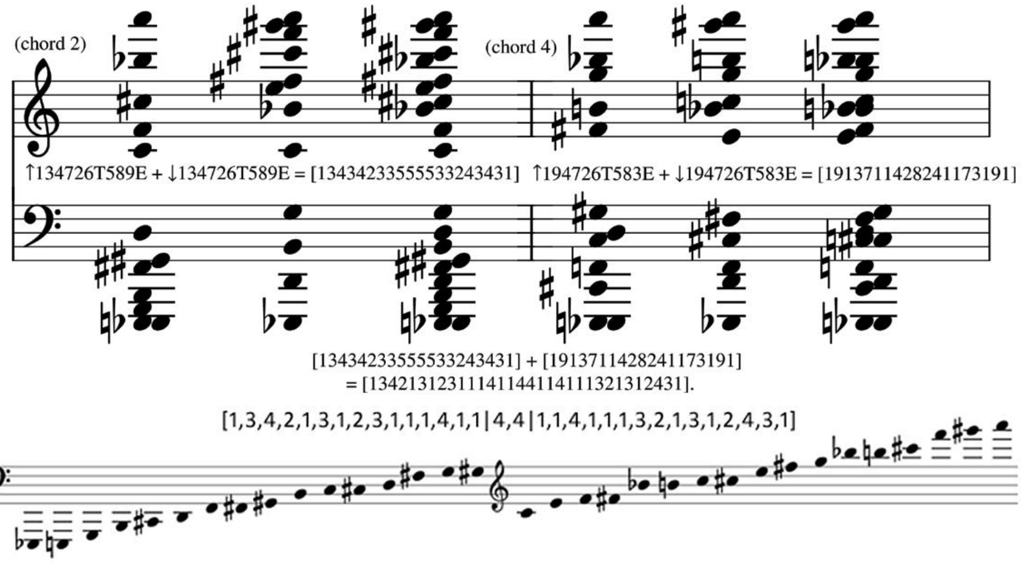


Figure 50: Prime scale; *Structural Declaration No.1* (2021)

The 34-pitch 62-STET (F1 to G6) OG scale is therefore:

Figure 51: OG scale; *Structural Declaration No.1* (2021)

In the catalogue accompanying John Hoyland’s first solo museum show, Hoyland’s abstract paintings are described as “structural declarations rather than romantic declamations” (Robertson, 1967), and the term “structural declaration” is similarly applied to this piece. In

68 However, Capuzzo points out that “the constraints that a given instrument imposes on Carter's favoured pitch resources is an important topic for future investigation” (2007, p. 105). Regarding P+I combinations, Bauer-Mengelberg and Ferentz discuss the inverted scale of the “permutation 3, 7, T, 1, 5, 2, 9, 4, 6, 8, E” (1965, p. 103), but similar to Carter, fail to combine (P+I) this all interval scale using common vertices. See Appendix II of this thesis for a 66-STET LIE scalic rendering of this particular scale.

this instance, the *Structural declaration* is derived from the directional scale step *sum* [↑+5↓-2↑+5] and vice versa: which pertains to Hoyland's practice of upending his canvases and painting from both ends.[69] In other words, counting up from the bottom of (and only using the pitches of) the prime scale, if E♭1 = 0, E1 = 1, G1 = 2 etc, the lowest tetrachord would be {E♭1 [↑5] D2 [↓2] B1 [↑5] G♯2}, and counting down from the other end of this scale, if A6 = 0, G♯6 = 1, F6 = 2 etc, the highest tetrachord = {A6 [↓5] B♭5 [↑2] C♯6 [↓5] E5}. If we take entry order out of the equation (although this is employed in the piece itself), the lowest tetrachord produced is [8, 3, 6] {E♭1, B1, D2, G♯2}, and the highest is [6, 3, 8] {E5, B♭5, C♯6, A6}. This structural approach is effectively a non-octaviated LIE scalic extension of George Perle's 12-tone "sum" identity. Perle suggests that,

> inversional symmetry, based on the unfolding of pitch classes through a single sum, is one of the two kinds of symmetry that you can have when you're dealing with a 12-tone scale. The other kind is based on the unfolding of pitch classes through a single interval, i.e. an interval cycle (Perle, 1992, p. 82).

Here, I am primarily/essentially dealing with combinatorial extensions of the former, and simply treating the extended FPF as an alternative step scale, in order to compositionally unfold the ramifications of the above structural declaration or sum. When compiled as ICs, the 9 possible ascending ↑ (up to) C4 ordered (+3+2+3) tetrachords and the inverse ↓ (down to) C4 (-3-2-3) variants are as follows:

[69] Hoyland can be seen (here) rotating a canvas in the BBC Arena documentary "*Six Days in September*" (C and C Art and Design Gallery, 2017).

↑ C4 (+3+2+3) tetrachords		↓ C4 (-3-2-3) tetrachords	
{**E♭1**, B1, D2,G♯}	[8, 3, 6]	{E5, B♭5, C♯6,**A6**}	[6, 3, 8]
{**E1**, C♯2,F2,B2}	[9, 4, 6]	{C♯5, G5, B5, **G♯6**}	[6, 4, 9]
{**G1**, D2, F♯2,C3}	[7, 4, 6]	{C5, F♯5, B♭5, **F6**}	[6, 4, 7]
{**B1**, F2, G♯3, C♯3}	[6, 3, 5]	{B4, E5, G5, **C♯6**}	[5, 3, 6]
{**C♯2**, F♯2, B2, D3}	[5, 5, 3]	{B♭4, C♯5, F♯5, **B5**}	[3, 5, 5]
{**D2**, G♯2, C3, F♯3}	[6, 4, 6]	{F♯4, C5, E5, **B♭5**}	[6, 4, 6]
{**F2**, B2, C♯3, G3}	[6, 2, 6]	{F4, B4, C♯5, **G5**}	[6, 2, 6]
{**F♯2**, C3, D3, G♯3}	[6, 2, 6]	{E4, B♭4, C5, **F♯5**}	[6, 2, 6]
{**G♯2**, C♯3, F♯3, C4}	[5, 5, 6]	{C4, F♯4, B4, **E5**}	[6, 5, 5]

Figure 52: Ordered tetrachords; *Structural Declaration No.1* (2021)

Everything within this registral span (E♭1↔A6) is of course symmetrical too (within and with regard to) the span itself, but the above ordering reveals other interesting symmetrical subsets, not least that the only tetrachords without a tritone [6] are centrally located at C♯2↑ and B5↓, an inverted and extended unfolding of the axial minor seconds (B3, C♯4) only present in the OG scale. The large scale ABA structure of this piece is also reliant on the 2-5-2 relationship (see Figure 53, below).

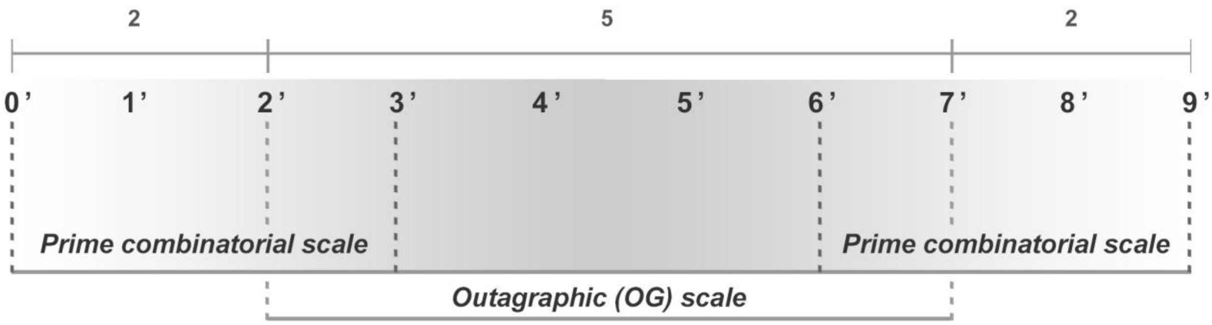


Figure 53: Large scale structure; *Structural Declaration No.1* (2021)

Various permutations of the ↑↓ 5, 2, 5 *sums* are shown in the opening bars (Figure 54):

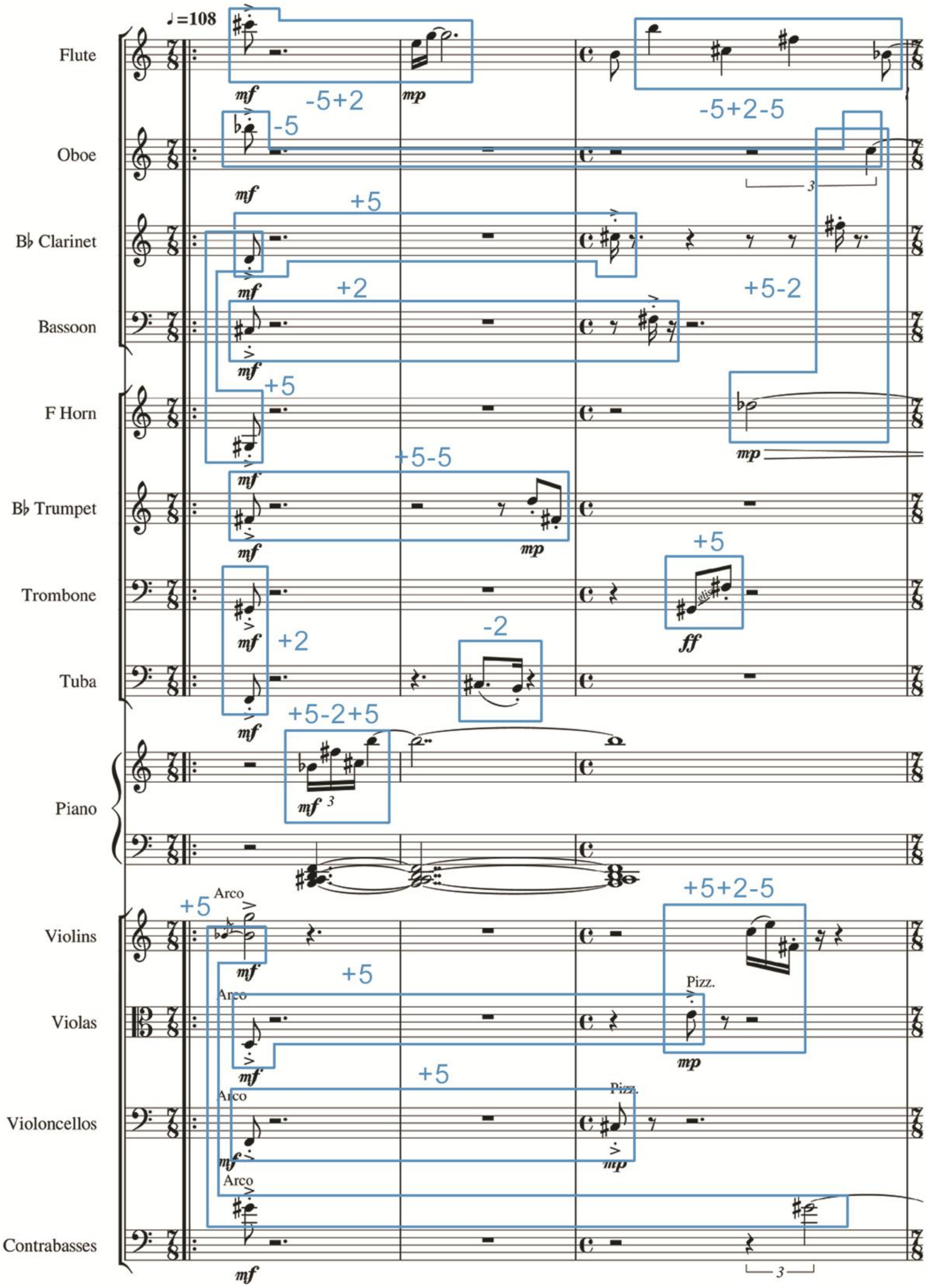


Figure 54: Annotated opening bars; *Structural Declaration No.1* (2021)

The introduction of the OG material at Letter B (bar 31) is emphasised by an underlying metric change that moves from 8 (4/4) to 7 (7/8) semitones, punctuated by staccato piano chords. Following on from my repurposing of Chopin's *Scherzo No.1* (previously discussed on pp. 83-84 of this thesis), at the temporal centre of this piece (bb. 75–77) is a scalic adaptation of Liszt's ascending dyads from his *Piano Concerto No. 1* (1830/1853), b. 19.

Figure 55: Scalic adaptation of Liszt's *Piano Concerto No. 1*

At night, my part of rural Carmarthenshire can be incredibly silent. One evening, whilst staring at the stars and contemplating how best to move between the sections of this piece, the only sound was a solitary cow sounding a slow protracted (mournful) F3 drone lasting a few seconds. This pitch and its symmetrical (from C4) *next of kin* counterpart (G4) is sounded in the horn part (Letter D, b. 60 in the score), and announces the bridge between the prime and OG scalic material of the middle section (Letter E to F, bb. 78–148).[70]

An argument against the validity of this whole LIE scalic endeavour might be that the extended structural concerns discussed here could be said to generally hark back to the 1920s, ostensibly ignoring the last century's academic musical concerns regarding timbral, spectral and micro/granular configurations, but as with the 1920s, the 2020s is a period of global socio-cultural upheaval where the choice between Marxism or fascism has generally transformed into one between eco-friendly humanism or populism/fundamentalism. That is not to say that LIE scales are analogous to a centralist political solution (they are hopefully far from it), but they are perhaps indicative of the need to explore alternative creative solutions that question the stability and viability of historically determinant structures. By choosing to score ostensibly tempered (yet abstracted and extended) chromatic music for traditional acoustic forces (pedagogical necessity aside), the composer is asking the orchestra/ensemble to become ironically complicit in the acknowledgement of its own demise, whilst potentially extending its contemporary repertoire (and therefore relevance), and simultaneously acting as a deceptive détournement, "a reminder that theory is nothing in itself, that it can realize itself only through historical action and through the historical correction that is its true allegiance" (Debord, 2014, p. 111). To paraphrase Marshall McLuhan (1964, p. 7), this medium was (and still could be) the carrier of an alternative message. I would argue that we are broadly still in a post (rather than uniquely Meta)-modern era, and alongside the obvious appropriation, this piece is also self-referential in

[70] The writing of this section coincided with the occupation of Kabul, August 15th 2021, and perhaps explains the general melancholic (legato strings) feel. I use the term 'melancholic' in the way that anything by (e.g.) Morton Feldman or Arvo Pärt might be described as such.

that it utilises and deconstructs various previously discussed devices. For example, the opening isolated punctual staccato chord is borrowed from *28 Occasions*…, and the pizz. string section at Letter B harks back to the punctualist ideas first introduced in *Outographic No.1*, which also uses the *step* scale (as both a concatenationist and architectonic device), and it is this that is specifically explored in my next short piece.

Queuing for gristle and bone (2022)
Scored for B♭ Trumpet and Piano
Duration: 4′ 04″

As lockdown restrictions were gradually lifted, many people hoped for a "new normal", a more sensible approach to global climate concerns. However, having grown accustomed to little traffic on the roads, resulting in less pollution and *no* litter, I was shocked to see the long queues outside a well-known "drive through" fast food establishment that heralded the return of both. The title of this piece is a direct reference to that queue, and all it stands for. This piece is built on an 87-STET complete piano keyboard scale (A0 to C8) framework, and the ↑↓ scale step displacement of chromatic dyads that expand from the central E4/F4 Y2 axis (see Figure 56, below).

Figure 56: Prime scale; *Queuing for gristle and bone* (2022)

Each dyad is numbered, and each dyad appears at least (and in many cases only) once in the piano part (only the octaviated 12th dyad is omitted). The 43-pitch extended (quasi-serial) entry order of the dyads is as follows:

Numbered Dyads:	13	26	29	4	30	19	32	9	34	11	0	25	37	2	38	17	15	40	5	42	43
Entry Order:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21

8	21	10	35	36	14	3	28	29	18	31	20	33	22	23	24	1	27	41	6	7	16
22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43

Figure 57: Numbered dyads; *Queuing for gristle and bone* (2022)

All the pages of the score with the original structural numbers accompanying the piano part are presented on the following pages with commentary. In brief, the dyadic piano pitches follow a registrally extended *Deutsch distribution-esque* rendering of the chromatic scale throughout. Deutsch found that subjects were unable to recognise familiar tunes "when the octaves were randomized" (1972, p. 412), and here the chromatic pitch entry order is similarly disguised. The first dyadic entries simply oscillate around the E/F axis, sounding the following directed ↑↓ OE chromatic path, a registral rather than timbral klangfarbonmelodie. Using the 87-STET scale (above) and the diagram below (Figure 58) as a reference, the six groups of pitch and interval related dyads are labelled A to F.

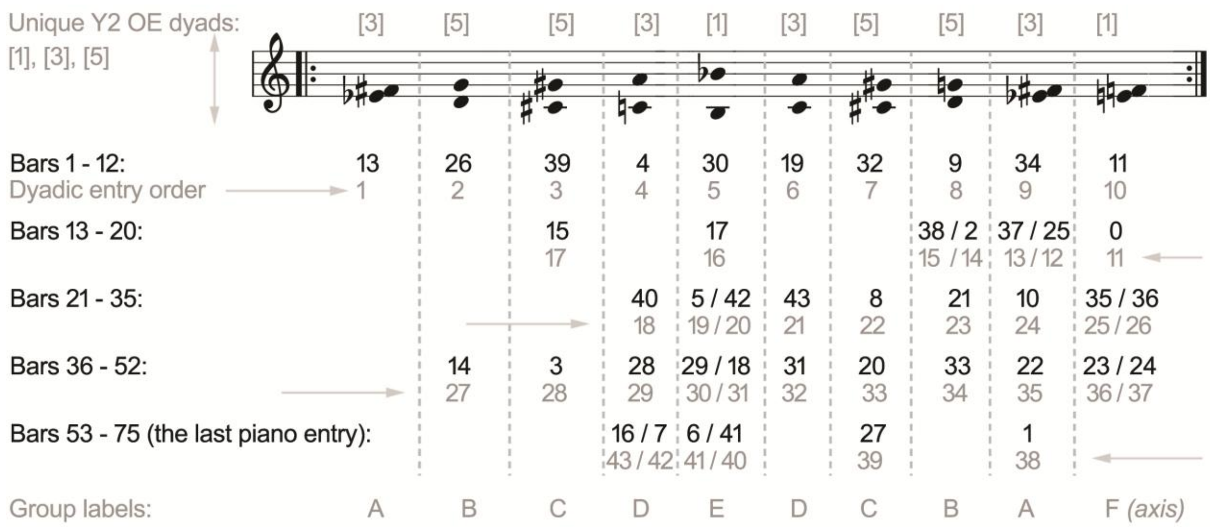


Figure 58: Dyadic entry order; Prime scale; *Queuing for gristle and bone* (2022)

These groups are utilised as temporal anchors, in that other material steps over (or around) them. This is similar to the rules of backgammon (or ludo), but mindful of Stravinsky's (get out of jail free card) suggestion that "to be perfectly symmetrical is to be perfectly dead" (1959, p. 20),[71] a certain amount of artistic license (or aesthetic preference) is involved. Broadly speaking, after entry, the numerically lowest (closest to the axial centre) dyadic pair of each group becomes the dominant pairing for that group, i.e. pair 1 in group A, 2 in group B, 3 in group C, and 4 in group D. For example, dyad no. 4 is repeated every 4 bars from bb. 9–72, whereas the other members of the D group (nos: 40, 28, 16, 7, 19, 43 and 31) are predominantly used only once. From bb. 39–71 only dyad no. 3 is repeated from the C group, every 3 bars (approx.), and dyad no. 2 from the B group is repeated every 2 bars (approx.). The first entry of no. 1 is at b. 52 and then in every bar up to the closing section (Letter J). In short, dyad no. 4 (a major sixth) is sounded every 4 bars, no. 3 (a fifth) every 3

[71] Combinations of both symmetrical and asymmetrical pitch collections are employed in *The Firebird* and *Le Sacre de Printemps* (Antokoletz, 1986, pp. 580–608).

bars, no. 2 (a fourth) every 2 bars, and no.1 (a minor third) is sounded in every consecutive bar. Does the knowledge of this numerical pitch/pulse relationship make it more perceptible? Either way, it should come as no surprise that dyad no. 5 (a major seventh) is deployed once every 5 bars (approx.) from Letters C–J. The only use of the E/F axial dyad (no. 0) heralds the first use of ornamentation using those two pitches exclusively (b. 13). When dyads 1–5 appear in the same bar, the hierarchical semiquaver (stepping stone) order of displacement is based on the original entry points, on a first come first serve basis, i.e. 4 first, then 2, 5, 3, 1; interesting examples of this are bars 39, 54, 55, and 69.

What is specifically being explored here (and essentially throughout this thesis) is the extent to which P+I extended chromaticism coupled with countable temporal intervallic *distance* might inform our perception of non-octaviated resolution/polarity. To this end I am ignoring Hindemith's (previously mentioned) cautionary suggestion that a system "without" the octave could be "adaptable for harmonic and melodic purposes... but it would be too clumsy for practical use" (1945, p. 25); not least that despite Hindemith's overt use of palindromic structures, (e.g.) *Ludus Tonalis* (1942/1963), where the *Postludium* (Part 25) is an exact inversion of the *Preludium* (Part 1), he asserted that "minor seconds and major sevenths have not attained full equality with the other harmonic intervals; and a thousand years of familiarity will not achieve it for them" (1945, p. 86), and that the tritone has "no definite significance, either harmonic or melodic" (1945, p. 89).[72]

[72] A LIE scalic 66-STET rendering of Hindemith's 1 and 2 series from *The Craft of Musical Composition (*1945) is discussed in Appendix II of this thesis.

Following on from previous experiments with *oddballs* and *chunks*,[73] the numerical and repetitive placement of the dyads in this piece is also an attempt to bring the idea of episodic memory into a temporal compositional strategy, using an approximated close-proximity principle.

According to Tulving "the essence of episodic memory lies in the conjunction of three concepts — self, autonoetic awareness, and subjectively sensed time" (2002, p. 5). When listening to music we are stimulated by sequential elements, and Healey *et al*. suggest that this contiguity is crucial to episodic memory, and arises from the encoding of an approximate "time scale invariant representation of temporal distance" (2019, p. 1). Although I am tempted to suggest that this might relate directly to the corporeal kit-drumming experience and a strict observance of tempi that might link auditory echo to our echoic memory systems, via an awareness of a temporal substrate (or distance), I will leave the description of episodic memory to Tulving, who points out that episodic memory is oriented towards the past, and has a "special, and unique, relationship to time" (2002, p. 6).

The trumpet part simultaneously sounds the (within range) OE piano pitches, and all trumpet ornamentations are constructed from the immediately preceding or surrounding material, e.g. see bars 9 and 10 in Figure 59:

[73] See structural notes for Symphony No. 1 (2020) in Appendix III

Figure 59: Opening bars*; Queuing for gristle and bone* (2022)

The trumpet and piano parts for this piece were recorded *as scored* by multi-instrumentalist Daniel de Gruchy-Lambert at his home studio.[74] In the performed score, dyad entry no.2 was misspelt in bars 16 to 20. Strictly (structurally) speaking, the sounded E♭ should have been a D, and this is corrected in the version below. From b. 26 the consecutive demisemiquaver substrate collections collapse inwardly by semitonal steps.

[74] Daniel also recorded the trumpet parts for my (more problematic regarding playability) piece *Fanfaronade* (2022), listed in Appendix III, and initially inspired my studio piece *16Hz The Threshold* (2020), see structural notes for this piece in Appendix III.

A
B♭ Tpt.
Pno.
B
B♭ Tpt.
Pno.
B♭ Tpt.
Pno.

Figure 60: Bars 11-30; *Queuing for gristle and bone* (2022)

From Letter F, the annotated version below (Figure 61) gives examples of how the surrounding (close proximity) pitch material is drawn together, informing the trumpet arpeggios in b. 36.

Figure 61: Bars 31-41; *Queuing for gristle and bone* (2022)

From Letter G, the call and response ornamentations between both parts are informed by the pitch material of the neighbouring dyads. The highest trumpet pitch D♭6 is sounded at

2’02” (b. 46), the temporal centre of the piece; and four bars later the only fast triplet is used.

Figure 62: Bars 42-62; *Queuing for gristle and bone* (2022)

The sparse texture of the trumpet part, for eight bars from Letter H, is juxtaposed with the density of the bars following Letter I.

Figure 63: Bars 63-68; Queuing for gristle and bone (2022)

From Letter J (Figure 64), the opening thematic ideas are reiterated. This piece closes with the solo trumpet echoing the two pitches of dyad no. 7 (a minor tenth {A3 [15] C5} extended inverse variant of no. 4 {C4 [9] A4}), over the demisemiquaver substrate with a 2/4 pulse. The use of the single oddball (B♭) pitch in b. 78 is a (tongue in cheek) means of expressively exploring closure without resorting to ritardando.

Figure 64: Bars 69-82; *Queuing for gristle and bone* (2022)

An Anthem for Libertatia (2022)
Part III: scored for large ensemble and choir (SATB) – *Duration: 7' 34"*

See scores in Appendix III for structural notes pertaining to Parts I and II

Having previously experimented with the limits of (i) extended chromaticism in *Queuing…*, and (ii) differing (often seemingly random) scalic combinations in many other pieces, to avoid heading into even more obscure (over complicated Heath Robinson-esque) generative territory, I next explored the ramifications of extending more traditional 12-STET octaviated and centrosymmetric scalic constructs. This piece and many of my subsequent larger scale pieces are increasingly Melakarta Raga-inspired.

Without dwelling on tuning, cultural significance, or connotations of appropriation (here the idiomatic borrowing is undertaken with respect), there are 72 numbered Carnatic Melakarta Ragas. These are also variously referred to as Janaka, Sampoorna, Raganga, and Mela ragas; and for convenience I will use the latter term. The 72 Mela are essentially the "parent" scales from which many thousands of variations might be constructed and are here discussed in relation to their closest ET PC equivalence, as similarly notated by Raja Ramanna (1995). The following diagram (Figure 65) is a useful means of grasping the generative symmetrical concept and the antipodal (or complimentary) OG nature of these heptatonic scales.

Within all Mela scales, the first pitch (e.g.) from "C" (Sa) {0} is fixed, and the notes of the lower tetrachord are chosen from the pitches C to F (Ma) or F♯ (Mi). With the upper tetrachord, G (Pa) {7} is fixed and the remaining pitches are chosen from G to the next upper C'. The first 36 numbered ragas are developed using F {5} as the fourth note, making {0, 5, 7} fixed, whereas the other 36 ragas (37–72) use F♯ {6}, and all these ragas therefore contain {0, 6, 7}.

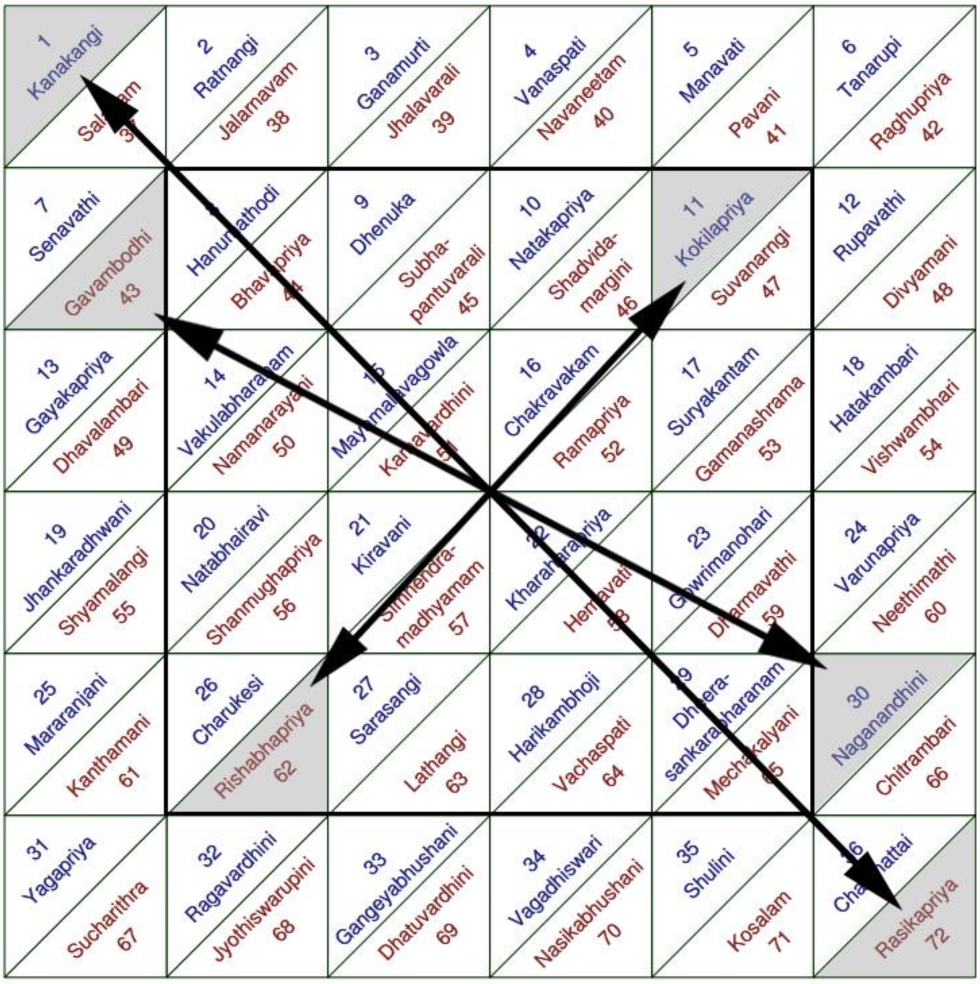
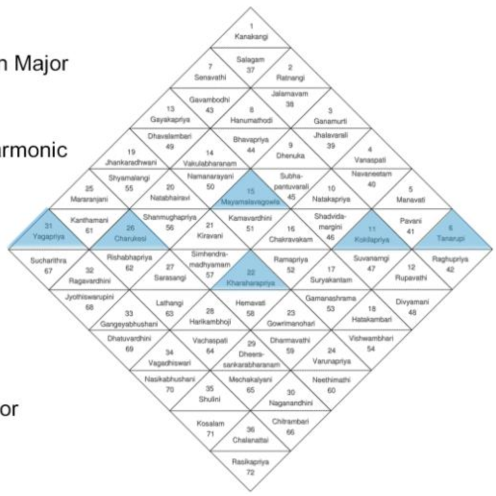


Figure 65: The Mukund Melakarta Raga Chart, based on Madhav, 2022

The remaining notes are used "in rising or falling order respectively and not more than two semitones appear consecutively" (Ramanna, 1995, p. 898). In the 6 x 6 (magic square) example above, the antipodal scalic pairs are all located in symmetrically opposite squares. The three paired examples I have depicted are nos.1 and 72, 11 and 62, 30 and 43. No.1 (= 1 + *0*) and no.72 (= 72 - 0), no.11 (= 1 + 10) and no.62 (= 72 - 10), no.30 (= 1 + 29) and no.43 (= 72 - 29). Leaving aside the pitches Sa and Pa {0, 7} that are common to all ragas, the complete (in this case minor sixth) pentatonic PC set for no. 72 {3, 4, 6, 10, 11} is the OG of no. 1 {1, 2, 5, 8, 9}, no. 62 {2, 4, 6, 8, 10} is the OG of no. 11 {1, 3, 5, 9, 11}, and no. 43 {1, 3, 6, 8, 9} is the OG of no. 30 {2, 4, 5, 10, 11}. Intervallic proportions equivalent to (and probably influenced by) those of the Mela Ragas crop up throughout western scalic systems,[75] and in this multi-cultural (universal) respect, LIEs might be thought of as centrosymmetric extensions of Chitravina N. Ravikiran's "Melharmony" (Morris and Ravikiran, 2006). By cross-referencing the PC sets listed by Morris and Ravikiran (2006, pp. 258–260) with the IC sets listed by Ring (2016),[76] it becomes apparent that there are only six centrosymmetric Mela scales, Nos. 6, 11, 15, 22, 26, and 31. These are all placed within the first/lower set of 36, they consequently avoid the tritone, include the *next of kin* fourth [5] and fifth [7], and are all intervallicaly, mathematically and geometrically symmetrical (see Figure 66, below):

Mela no:		Mela name:	IC Structure:	Western Scale Name:
1	5	Kanakangi		
2		Ratnangi		
3		Ganamurti		
4		Vanaspati		
5		Manavati		
6		Tanarupi	[1, 1, 3, 2, 3, 1, 1]	
7	4	Senavati		
8		Hanumattodi		
9		Dhenuka		
10		Natakapriya		
11		Kokilapriya	[1, 2, 2, 2, 2, 2, 1]	Neapolitan Major
12	3	Rupavati		
13		Gayakapriya		
14		Vakulabharanam		
15		Mayamalavagaula	[1, 3, 1, 2, 1, 3, 1]	Double harmonic
16		Chakravakam		
17		Suryakantam		
18		Hatakambari		
19		Jhankaradhvani		
20		Natabhairavi		
21		Kiravani		
22		Kharaharapriya	[2, 1, 2, 2, 2, 1, 2]	Dorian
23	3	Gaurimanohari		
24		Varunapriya		
25		Mararanjani		
26		Charukeshi	[2, 2, 1, 2, 1, 2, 2]	Major Minor
27	4	Sarasangi		
28		Harikhamboji		
29		Dhirasankarabharana		
30		Naganadini		
31		Yagapriya	[3, 1, 1, 2, 1, 1, 3]	
32	5	Ragavardhani		
33		Gangeyabhusani		
34		Vagadhibhusani		
35		Sulini		
36		Chalanata		



= centrosymmetric mela scales

Figure 66: The six centrosymmetric Mela scales, Nos. 6, 11, 15, 22, 26, and 31

[75] For example, no.8 = Phrygian, 9 = Neapolitan Minor, 11 = Neapolitan/Lydian Major, 15 = Double Harmonic (Enigmatic), 20 = Aeolian/Minor, 21 = Harmonic Minor, 22 = Dorian, 23 = Melodic Minor, 26 = Major-Minor, 28 = Mixolydian, 29 = Ionian, 57 = Double Harmonic Minor, 65 = Modern Lydian, 70 = Hungarian Major.

[76] Ian Ring's list of numbered Carnatic Melakarta can be found here (Ring, 2016).

Libertatia is generated from extensions of Mela scales no. 26 (Major-Minor) and no. 22 (Dorian).

Composers notes:

(i) The use of marimbas, vibes and "fixed pitch" scales

(ii) The phonetic use of text

(iii) Etymology and reasoning behind the title - Libertatia

(i) The use of marimbas, vibes and "fixed pitch" scales

Regarding the psychophysical spectral vs. temporal (autocorrelation) models of pitch perception (Yost, 2009), I favour the latter and it is the short (fast attenuated) percussive (yet tuned) attack derived from stick/mallet use that I find interesting. The timbre of the marimba signifies its wooden lineage from log drum to tongue drum, to balafons (xylophones are perhaps more indicative of chime stones per se). Similarly, vibes could be said to descend from bells (once cast, a bell's tuning is of course ostensibly *fixed*), and perhaps it is the percussive attack combined with the fixed (or absolute) pitched nature of these instruments that lends itself to the realisation of "fixed pitch" scales? My interest in composing for the marimba also follows on from my work with master balafonist N'Famady Kouyate,[77] and the relative success (in that it received BBC radio airplay) of my algorhythmic piece for marimbas, *Fractal #1* (2016).[78]

[77] N'Famady is a master balafonist from Guinea based in Wales. His input was intrinsic to the *Gwaith Cartref* album I produced in 2020, and I spent many days listening to and mixing the balafon parts he recorded for the track Balafon Groove.
[78] This piece was aired by Adam Walton on BBC Radio Wales (July 9th 2016), and can be heard here.

(ii) The phonetic use of text

In this piece the text is the title and the title is used both phonetically and holistically; i.e. a principle of homeopathy is that dilution *increases* potency. If so, might the repetition of a single word or phrase carry a similar potential? At what point does a chorus (a song with no verse) become a chant or mantra (and vice versa)? In short, this is not a text setting (as such), nor does it allude to being a libretto. [79]

(iii) Etymology and reasoning behind the title - Libertatia

On September 24th 2020, the UK Department for Education published guidance for school curriculums which effectively banned the study of any anti-capitalist movements or literature. Buried under the heading *Plan your relationships, sex and health curriculum*, it stated that "Schools should not under any circumstances work with external agencies that take or promote extreme political positions or use materials produced by such agencies" (Department for Education, 2020). Following a Guardian newspaper article, examples of these *positions* were subsequently revised, but the original version listed "a publicly stated desire to abolish or overthrow... capitalism" (cited in Busby, 2020) as one of them. While contemplating the list of left-leaning artists that might potentially be inappropriate to study (perhaps a large percentage of the avant-garde), I discovered *The Pirates' Who's Who* (1929) by Philip Gosse, which mentions the socialist colony known as Libertatia (or Libertalia). According to Gosse, the *ideal* colony of Libertatia was "run on strictly socialistic lines, for no one owned any individual property; all money was kept in a common treasury, and no hedges bounded any man's particular plot of land" (1929, p. 219). Gosse's section on Captain

[79] I had explored this idea in previous work, e.g. *Bethlehem SA19 6YH* (2014) and *Plenty of Furniture* (2014), following many decades of chanting *Nam-Myo-Ho-Renge-Kyo* and learning how to harmonise vocally.

Misson and Libertatia (1929, pp. 212–219) also mentions the emancipation of slaves and the use of a white (rather than traditionally black) pirate flag; with the motto "For God and Liberty" embroidered on it, rather than the skull and crossbones. William Burroughs mentions Libertatia in *Cities of the Red Night* (1981, p. 11), and as here, "historical veracity is decidedly not the point…; Burroughs is more concerned with producing different historical potentials" (Houen, 2006, p. 532). However, as musicians we epitomise the societal synthesis of work and play, and this privilege necessitates that we at least contemplate potential universalities and egalitarian utopias.

Composer Alexander Schubert defines his personal methodology as a contradiction between the opposing poles of (amongst other things) *conception* and *intuition* (2020, pp. 14–16), and as much as I would like to think that the combination of both renders my work as being somehow politically allegorical with a tinge of metaphysical potential, can any abstract collection of sounds be perceived as such outside of an associative descriptive framework? More specifically, what might *instrumental* music of, for, or by the precariat sound like? Since working in a "noisy" (yet rhythmically intriguing) factory in the late 1970s, I have pondered what direct effect the industrial revolution might have had on folk music. To what extent might factory workers have either yearned for a pastoral idyll, or raged *against* the machine? Personally, I have tended towards the latter. In this respect, my music is perhaps related to an overtly political wing of (so called) "New Conceptualism", spearheaded by Johannes Kreidler (Erwin, 2016). Before such times as the only music designated for study purposes becomes officially *prescribed*, we should embrace all alternatives. This particular exploration of *Libertatia* began its generative life as follows.

From an axial 'C' pitch, Hauptman's (previously mentioned) harmonic dualist ↑↓ [3, 4|4, 3] pentatonic = the 14-STET scale {F, G♯, C, E, G}. Expanding (x2) the intervals of this set [3, 4, 4, 3] by what Schillinger calls the "optical projection through extension of the ordinate" (1943, p. 244), and what is mathematically known as *homothetic transformation* through *scalar multiplication*, produces [6, 8, 8, 6] {B♭ [6] E [8] C [8] G♯ [6] D}. Combine the pitches of both scales and we arrive at the OE heptatonic set {C, D, E, F, G, G♯, B♭} [2, 2, 1, 2, 1, 2, 2] = Major-Minor scale, aka the 26th Mela raga (see Figure 67, below).

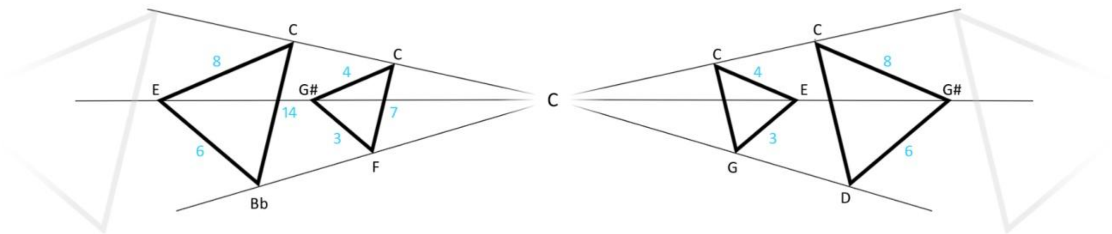


Figure 67: A visual representation of "scalar" expansion

By similarly doubling then combining the inverse (and more consonant sounding) 14-STET scale {F, A, C, E♭, G} [4, 3|3, 4] to [8, 6|6, 8] we get {B♭, F♯, F♯, D} + {F, A, C, E♭, G} = the octatonic OE set {C, D, E♭, F, F♯ G A B♭} [2, 1, 2, 1, 1, 2, 1, 2], also variously known as the Blues Scale II (mentioned in Part 1 of this thesis).

Libertatia as a whole/trilogy is composed in ABA ternary form. Part III imitates (and embellishes) much of the material introduced in Part I, with the addition of vocal parts that are informed by various structural concerns. The title is used phonetically (*Li-ber-ta-ti-a*), and the punctual staggered vocal delivery that begins at Letter D is contrapuntally treated at

Letter E (b. 71) using the Blues scale II, and partly inspired by The Trashmen's *Surfin Bird* (1963).

Figure 68: Punctual vocal entries; *Libertatia, Part III* (2022)

Letter F, the Zenith of this piece as a whole, is derived from considerations of axial extensions; (i), as used in Bartók's *Piano Concerto No. 2 (*1930/1931) - Movement II, (ii), as discussed by Bernhard Ziehn and (iii), those defined by Sigfrid Karg-Elert's twelve minor/major *Ursprungslagen* or *source positions* (Byrne, 2018, p. 70).

The opening adagio section from the second movement of Bartók's Piano Concerto No. 2 (1930/31) is an archetypal example of parsimonious centrosymmetric construction, derived from quintal harmony, with two stacked fifths either side of a fluctuating (at times overlapping) central span, i.e. [7, 7, ?, 7, 7]. The colour-coded boxes added to the score below (see Figure 69) allude to various temporal symmetric structural devices, and the annotated numbers refer to the harmonic intervallic distance used for the central axis [7, 7,-?, 7, 7].[80] Of interest are the axial ICs being here given a minus sign; aside from one incidence of minus 2 in b.7, these only appear in pairs, and always a crotchet apart, i.e. bars 5, 12 and 14. If the trichords {C3, G3, D4} [7, 7] + {D4, A4, E5} [7, 7] = the pentatonic scale {C3, G3, D4, A4, E5}, then the central axial 'D' has a zero value = [7, 7 (0) 7, 7]. Therefore (in b.7, see Figure 69, below) the simultaneous sounding of the trichords {C3, G3, D4} [7, 7] + {C4, G4, D4} [7, 7] can be thought of as [7, 7, (-2) 7, 7].

Place (or picture) two hands palms down on a piano keyboard with both thumbs on D4, then symmetrically move both hands inwards, and watch how they overlap. Numbers aside, this simple visual representation of axial-derived harmonic distances transcends negative connotations. Bernhard Ziehn was perhaps the first theorist to mention symmetrically centralised cognition related to the visual layout of a piano keyboard, namely that "any tone may serve as a centre, but from d only we receive relations simple and clear" (1912, p. 1). In other words, 'D' and its OE inversional compliment 'A♭/G♯' are the only visually centric piano keys; of course any of the 12 may function as an axis. As Busoni was both a student of Ziehn (Nolan, 2003, p. 223) [81] and Bartók's mentor (Hooker, 2005, p. 285), one might speculate that Bartók was familiar with Ziehn's "generalized conception of combinatorial space" (Nolan, 2003, p. 223).

[80] Bartók's registrally dispersed axial intervals are symmetrically ordered as follows: bb.1–3; [7, 3, 7] [10] [7, 3, 7], is followed by bb.3–4; [7, 1] [5, 1] [3, 7] which is the inverse of [1, 7] [1, 5] [7, 3] found in bb.15–17, bb.5–7; [-2, 7, 10, 7, -2], bb.8–9; [12, 10, 12] bb.11–12; [2, 5, 3] is repeated at bb.21–22, and b.20 contains [3, 5, 3]. For symmetry in Bartók see Lendvai (1971), also Cohn (1988) and Bernard (1986).

[81] Busoni's *Fantasia Contrappuntistica* (1910) is thought to have been influenced by Ziehn, and as with Ives, the spectre of Busoni hovers over this entire LIE scalic endeavour.

Figure 69: Intervallic analysis of Bartók's Piano Concerto No. 2, Movement II (1930/1931)

Discussing the "inversion of intervals" in his *Manual of harmony...* (1907), Ziehn states the obvious point that "if the lower tone of an interval is placed an Octave higher, or the upper tone an Octave lower, the interval is *inverted*. A Second becomes a Seventh, a Third a Sixth, a Fourth a Fifth, and vice versa" (1907, p. 2), but Ziehn's examples are essentially still conceptualised as being unidirectional, from the bottom up *or* the top down rather than P+I centrosymmetric (with *overlapping* intervals; visually analogous to a *crossing* of hands). If we apply Ziehn's definition of (what is ostensibly Zarlino's *principale* and *replica*) inversion as described above, then the major third (e.g. C5-E5) becomes an extended 20-STET Lie scale {E4---C5-E5---C6} with a (reduced to 'D') Y1 central identity, and the fourth (e.g. C5-F6) can be rendered as a 19-STET scale with a Y2 (D/E♭) identity {F4---C5-F5---C6}. Although *limited*, this particular method of tetrachordal expansion from dyads, that relates spans to ICs, is an example of why registrally extended scales are potentially rich generative veins.

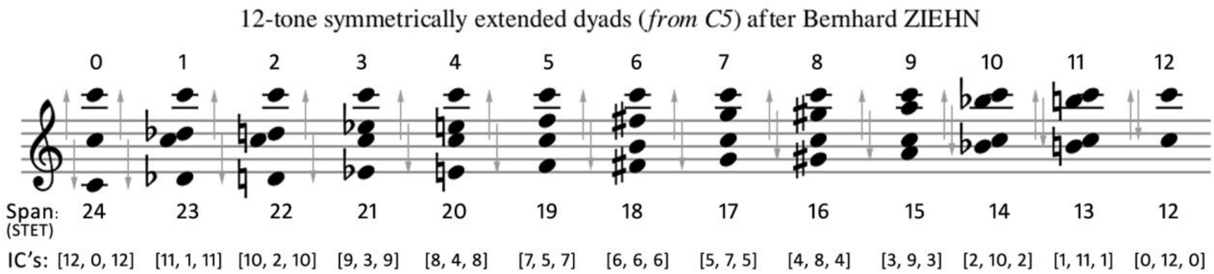


Figure 70: Symmetrically extended dyads based on Ziehn, 1907, p. 2

All of the above concerns might be said to inform my methodology in general, but as Part III of *Libertatia* was to be the only example of a choral (SATB) in the portfolio, these concerns seemed to take on more significance, and at Letter F, Karg-Elert's twelve minor/major *Ursprungslagen were a* specific influence (see Figures 71 and 72, below).

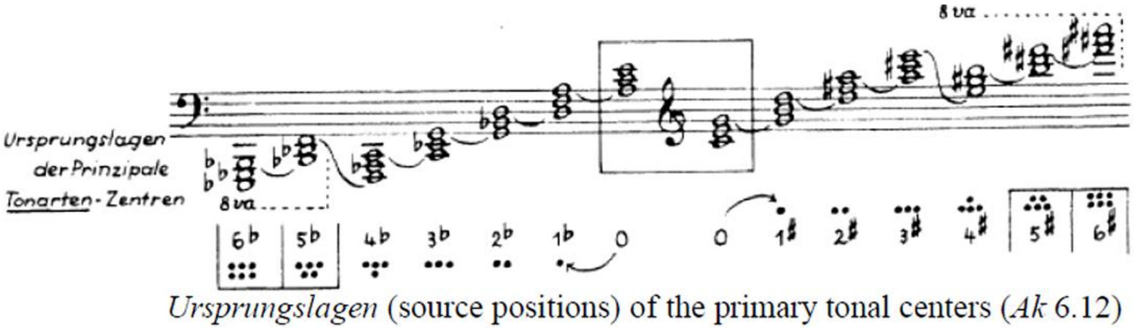


Figure 71: Karg-Elert's *Ursprungslagen* diagram, based on Byrne, 2018, p. 123

By combining (and extending-completing) Karg-Elert's symmetrical ↑↓ chordal pairs (i.e. 1 = Am + CM, 2 = Dm + GM, 3 = Gm + DM…), the *complete* set of 12 (if 13 = 1…) is as follows:

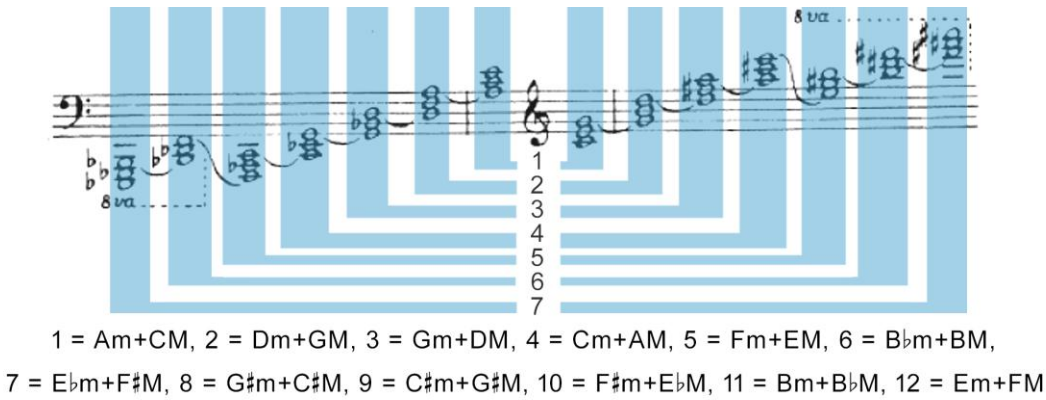


Figure 72: Extension of Karg-Elert's *Ursprungslagen*

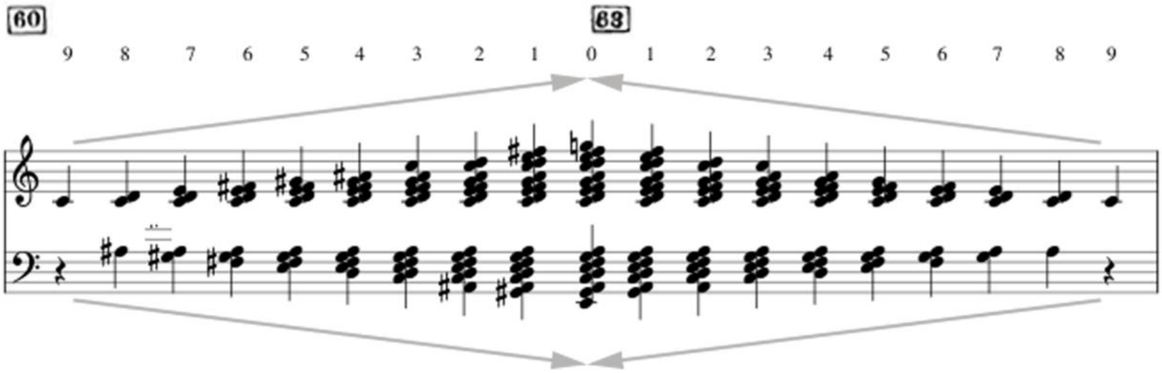
Using an axial 'G♯' and including the 'D' vertices, various pentatonic, hexatonic, and heptatonic pitch collections are produced (see columns 1 and 2 in Figure 73); add Bartók's quintal harmony, i.e. two stacked fifths either side of a fluctuating central span (see column 3), and these are the generative scales used from bb. 81–104, which harmonically move from a suspended pentatonic scale (No. 1) to the 'D' Dorian 22nd Mela scale (No. 12), via the 6th Mela scale (No. 4) and the Double Harmonic 15th Mela scale (No. 10).

	Ursprungslagen + D	*IC's*	*Central Chord + Stacked Fifths*	*Entry bar no.*
1	{D,E,G,A,C,D}	[2,3,2,3,2]	{F,C - G,C,D,E,A - E,B}	b. 81
2	{D,F,G,A,B,D}	[3,2,2,2,3]	{E♭,B♭ - F,G,A,B - F♯,C♯}	b. 85
3	{D,F♯,G,A,B♭,D}	[4,1,2,1,4]	{C,G - F♯,G,A,B♭ - F,C}	b. 97
4	{D,E♭,E,G,A,C,C♯,D}	[1,1,3,2,3,1,1]	{G,D - A,C,C♯,D,E♭,E,G - D,A}	b. 99
5	{D,E,F,G♯,B,C,D}	[2,1,3,3,1,2]	{D,A - E,F,G♯,B,C - G,D}	b. 101
6	{D,E♭,F,F♯,B♭,B,C♯,D}	[1,2,1,4,1,2,1]	{C♯,G♯ - E♭,F,F♯,B♭,B,C♯ - G♯,E♭}	b. 103
7	{D,E♭,F♯,B♭,C♯,D}	[1,3,4,3,1]	{C♯,G♯ - E♭,F♯,B♭,C♯ - G♯,E♭}	b. 103
8	{D,E♭,F,G♯,B,C♯,D}	[1,2,3,3,2,1]	{C,G - D,E♭,F,G♯,B,C♯,D - A,E}	b. 105
9	{D,E♭,E,G♯,C,C♯,D}	[1,1,4,4,1,1]	{C♯,G♯ - E♭,E,G♯,C,C♯,D - G♯,E♭}	b. 107
10	{D,E♭,F♯,G,A,B♭,C♯,D}	[1,3,1,2,1,3,1]	{C♯,G♯ - E♭,F♯,G,A,B♭,C♯ - G♯,E♭}	b. 109
11	{D,F,F♯,B♭,B,D}	[3,1,4,1,3]	{E♭,B♭ - F,F♯,B♭,B - F♯,C♯}	b. 111
12	{D,E,F,G,A,B,C,D}	[2,1,2,2,2,1,2]	{E♭,B♭ - F,G,A,B - F♯,C♯}	b. 112

Figure 73: Generative scales used in *Libertatia, Part III* (2022)

Towards the close of Part III, the restatement of the Vivaldi-inspired strings (first introduced in Part I) announce the finale/chorus, which climaxes with a high 'A5' in the soprano part (b. 228). This section is also inspired by the symmetrical unfolding of Ives's *Psalm 90* (1896–1901), but whereas Ives thwarts symmetricality by not venturing to the symmetrically required high 'G♯', with *Libertatia* the harmonic unfolding is completed (or resolved). Ives was clearly developing integral (or total) pitch and pulse organisational methods prior to the pitched serialism of the second Viennese school; overt examples of Ives's palindromic usage (Schoffman, 1981) include *Three Harvest Home Chorales* (1898?-1912), *Soliloquy* (1916/17), and *On the Antipodes* (1922/23). In verse 9 of Ives's *Psalm 90*, the symmetry of the whole tone scale (see Figures 74 and 75, below) ascending and descending from an axial 'C' is only broken at the zenith point (bb. 62–63) by the replacement of a G♯ with a G natural. This might be interpreted as: (i) a passing, voice leading or cadential pitch, (ii) a means of not forcing the upper soprano vocal range, (iii) a way to emphasize the word "Wrath", or (iv) simply breaking the symmetry in order to avoid what would be an inverted tritone, i.e., the expulsion of the diabolus.

Figure 74: Score extract from *Psalm 90* (Ives, 1896)



The wholetone scale (minus G♯5) unfolds from a C4 axis
(*the integer sequence also refers to semiquaver durational values*)

Figure 75: Vocal part structure; *Psalm 90*, *Verse 9* (Ives, 1896)

Rather than an afterthought, the choral parts intentionally only appear in the third section of *Libertatia*. If one listens to the complete trilogy consecutively, then, after having listened to Part III, listen again to Parts I and II, and one should be able to internally hear the various unscored vocal refrains from Part III. As well as involuntary musical imagery (INMI, or *earworms*), this plays with the concept of echoic and episodic memory, if not, ultimately, indirect realism.

Symphony No.2 (2022)
Scored for 1 picc., 3 fl., 2 ob., cl., b.cl., bsn., cbsn., 2 f.hrn., tpt., tbn., b.tnb., timp., sn., cym., guit., pno., synth (pre-recorded or midi standard), vln., vla., vc., cb
Duration: 21′ 15″

The 15th Mela scale (or Double Harmonic) [1, 3, 1, 2, 1, 3, 1] is used as a generative source scale for this piece, which, ascending from E♭ (with a silent axial 'A') = {E♭, E, G, G♯, B♭, B, D, E♭}. Discovering that this scale was also popularly known as (and harmonically equivalent to) the "Flamenco mode", informed the inclusion of a classical guitar. I discussed the performance implications of performing sextuplets at 124 BPM with various guitarists,[82] and the general conclusion was that this part was potentially playable by an expert (see opening two bars, Figure 76).

Figure 76: Guitar part; *Symphony No.2* (2022)

For an example of what I regard as virtuosic *prestissimo* timing, listen to some of the numerous online performances of the first movement from Joaquín Rodrigo's *Concierto de Aranjuez* (1939), then listen to Paco De Lucía's impeccable rendition (Zsivera, 2013).[83] In *Symphony No. 2* the guitar part is often simply an octave doubling of the mallet synth substrate that enters at b. 13, fades in and out throughout the piece, and underpins the differing scalic combinations.

The synth part might be either pre-recorded, or aired in real-time using a Midi standard Mallet Synthesiser (Square Lead Expr.). In both cases this necessitates that a conductor, leader or percussionist is provided with a click track. If the much-discussed human tendency for anticipating stimulus onsets known as "negative mean asynchrony" (NMA) (Repp, 2005, p. 973) is not universal (as was previously thought) and is "largely affected by the cultural origin of participants" (Lagarde, 2021, p. 8), then the use of click tracks and temporal

[82] Notably Tom Spencer (*TMTCH* and *The Professionals*), who also suggested that sections of this piece sounded like Dukas's *Sorcerer's Apprentice* (1897) used in the Disney film *Fantasia* (1940). Ironically, *Fantasia* was the first film I ever saw (at the cinema) in the mid 60s; we didn't have a TV.
[83] This piece can be heard here (Zsivera, 2013).

substrates towards a fine-grained quasi-Leibnizian adherence to tempi (see Bizzell-Browning, 2024) is arguably an egalitarian pursuit.

In this piece the demisemiquaver (4) element of the underlying 6:4 hemiola is first introduced by the woodwind, brass and piano (bb. 5–8), but it is the underlying bass and cello parts that give depth to this idea. The first cello demisemiquaver ostinati entry is at Letter E (b. 54), and from Letter I (b. 93) the solo bass part (see Figure 77, below) replaces the synth substrate with a 48 bar (incl. repeats) ostinati that gradually swells while still exclusively sounding the pitch material of the E♭ 15th Mela scale. Following input from various contrabass players (primarily Richard Gibbons of the BBC National Orchestra of Wales) this part was deemed to be *generally* playable. Dittersdorf's *Double Bass Concerto* (1762) was also referential regarding the speed of semiquaver delivery in bass and cello parts, as was Henze's *Symphony No. 7* (1982–83).

Figure 77: Contrabass ostinati; *Symphony No.2* (2022)

At Letter D (b. 46) a melodic theme (one of two) is introduced by the cello; this refrain acts as a scalar (half-time) triplet reminder of a sextuplet pulse, and the purpose of both refrains is to highlight the movement between differing scalic materials at later stages (note the additional 'F' pitch).

Figure 78: Cello refrain, first theme; *Symphony No.2* (2022)

Variations of the first cello refrain appear in the woodwind (Letter F, b. 63), the horn (b. 241), and the flute (b. 314), at which point both themes begin to coalesce. The second theme first appears in the oboe (Letter K, b. 133, see Figure 79, below), and is largely a foil for the substrate, and a tool for extending the piece itself.

Figure 79: Second theme, oboe refrain; *Symphony No.2* (2022)

At Letter J (b. 117) an isolated axial 'A' pitch is first sounded in the viola part, which is further embellished (b. 123) by a 10-STET heptatonic variant of the E♭ (prime) Mela {E, F, G♯, A, B♭, C♯, D} = [1, 3, 1, 1, 3, 1] that includes both the pitches 'F' and 'A'. At Letter K (b. 133) the substrate reverses direction harmonically, returns to the prime scale, and expands from a 10-STET to a 20-STET span that underpins the development of the second refrain. At Letter L (b. 185) the bass ostinati is reiterated, then at Letter N (b. 209), nearing the temporal centre of the piece (9' 37"), the prime E♭ Mela scale and its tritonal transposition {A, B♭, C♯, D, E, F, G♯, A} are superimposed, resulting in a decatonic 'E♭ + A' scale {E♭, E, F, G, G♯, A, B♭, B, C♯, D, E♭}; which includes the 10-STET heptatonic scale and adds the augmented triad {F, A, C♯} to the original 15th Mela, while also sounding the axial 'A'. The only possible whole tone scale within this E♭ + A collection is {E♭, F, G, A, B, C♯}, with a silent ‘A’, the resulting pentatonic 8-STET (minor sixth) scale = {B, C♯, E♭, F, G}. This particular scalic permutation is introduced at Letter P (b. 236), followed by a reiteration of the decatonic collection at Letter Q.

The temporal “golden” section of this piece (21’ 15” ÷ 1.618 = 13’ 13”) falls at Letter R (approx.), where the semiquaver bass and cello pattern is truncated, allowing the snare and timps to carry both pitch and pulse. The four tuned and fixed (used throughout) timp pitches {G2, B2, D3, E♭3} from the E♭ Mela are spread over 28 (7 x 4) rhythmic *occasions of experience*, where the quaver based timp phrase has the same duration as two semiquaver snare patterns. Both the timp and snare parts are rhythmically centrosymmetric (Messiaen’s *non-retrogradable* rhythm), and both of these parts utilise what I have called the “rubato” rhythm (Figure 80); timbrally inspired by Ravel’s *Boléro* (1928), numerically by Ives, cognitively by simple whole number consecutive distance counting (+1, +2…+7), and (in this instance) conventionally by also sounding the first downbeat of each crotchet in the snare, to encourage entrainment.

Figure 80: Rubato rhythm; *Symphony No.2* (2022)

At Letter S (b. 308) the timps settle on a constant ‘G’ pitch, and the centrosymmetric rhythm is adjusted to span eight crotchets (two bars). This forms the ground for the pizz. cello solo (adapted from, and informed by, Michael’s cello rendition of my *Notes of Protest*), which along with contrapuntal allusions to the main theme(s) in the winds and brass, utilises the E♭ Mela scale. Letter W (b. 408) introduces a (personal signature) punctualist section, and the piece closes with reference to the opening flute triplets (Letter A), and a *pianissimo* and transposed piano version of the synth substrate first heard at b. 13.

An Apology To The Ocean (Ymddiheuriad I’r Cefnfor) (2023)
Scored for Tenor Saxophone and Trombone
Duration: 6′ 16″

This piece was initially composed for the Welsh section of the International Society of Contemporary Music (ISCM) World Music Day 2024. The structure of this piece is inspired by (i) oceanic wave motion, where “individual waves move at twice the speed of the group” (Biello, 2006), but are bound by shared energy, (ii) my discovery of a correspondence between the Dorian scale (AKA the 22nd Mela Raga) and Robert ap Huw's “strange tuning” (Dart, 1963, p. 58) scale, and (iii) the alternating stressed couplets of Cywydd Deuair Hirion, the 10th Welsh poetic metre (see GPC Online, 2014).

Robert ap Huw's Welsh manuscript is (perhaps) the earliest surviving example of harp music in Europe. Thurston Dart discusses Ap Huw's “strange tuning... y kower chwith, or dierth” (1963, p.58) scale, where the “entire instrument was tuned in alternating steps of a tone down and a fourth up... A G C B♭ E D G F B♭ A, etc.” (ibid). In ascending order, this scale {G A B♭ C D E F G...} [2, 1, 2, 2, 2, 1, 2] corresponds to the *Dorian* scale. The scale is centrosymmetric, as is the structure of an oceanic wave set (or group) which “typically consists of... larger waves in the middle... because waves in the rear tend to move forward, build in size and then diminish as they reach the front. Although individual waves move at twice the speed of the group, they are bound to it by the energy they all share" (Biello, 2006), and in this piece the sax lines are scored at double the speed of the trombone. The rhythm of this piece is derived from the Cywydd deuair hirion (the 10th codified Welsh poetic metre) and is constructed from 7 syllable lines (see; GPC Online, 2014). The alternating couplets place the dynamic stress/accent as follows: x x x x x x X (trombone), x x x x x X x (sax - double time). These accents are further informed by the rhythm of Ap Huw's "tresi heli... 1 0 0 0 1 1 1 | 0 0 0 1 0 1 1" (Dart, 1963, p. 60).

The large scale structure is in arch/ternary ABA form, where the A sections explore the prime {G A B♭ C D E F G} scale, and the central B section uses what Schillinger calls the "optical projection through extension of the ordinate" (1943, p. 244). In other words, the scale underpinning the central B section is intervallicaly doubled; this is mathematically known as *homothetic transformation* through *scalar multiplication*. From a central C♯ axis, [2, 1, 2, 2, 2, 1, 2] becomes [4, 2, 4, 4, 4, 2, 4] resulting in the fixed-pitch hexatonic set {G A B C♯ E♭ F G...}.

The opening A section (bb. 1–23) is synonymous with a *natural* order, the B section (bb. 23–41) represents temporary human intervention, while the closing A section (bb. 42–65) signifies a *natural* return, a balance.

Although I did not listen to any *early music* before or during the compositional process, the specific nature of my scalic research for this piece was clearly an aesthetic influence on the outcome. In personal correspondence, John Croft suggested that the "cantus-like trombone" reminded him of “some renaissance music for cornetts [e.g.] *Upon La Mi Re* by Thomas Preston” (Croft, 2024). In short, *An Apology…* demonstrates that LIE scales can be used as a generative source for a broad range of musical genres and sound worlds. Steven Mai (a member of *The World Youth Orchestra*) recorded the trombone part *as scored* in (almost) one take, which gave us time to experiment with glissandi, notably bb. 21–24; where the piece moves into new scalic territory. For Steven, the larger intervallic leaps between pitches were the only problematic elements. Naomi Bayley recorded the sax part a few days later.

Biting the Bullet between Gritted Teeth (2024)
Performed by Matt Baker (Harmonica), Richard Gibbons (Bass) and myself (drums)
Duration: 6’ 20”

Matt, Richard and I have all intermittently performed with ‘The Low Down Dirty Dog Blues Band’, based in Wales. By removing the guitars and vocals from this (particular) *Blues* sound world, one is left with (i) the solid/stable rhythmicity of the bass and drums and (ii) Matt’s harmonica improvisations. These two auditory elements are here deployed over a (non-blues) 7/8 time signature, which hopefully makes this piece more genre-unspecified; and along with the *limited* harmonica scale, potentially more challenging and rewarding. When playing the Blues, a ‘C’ harp would generally be used over a (2^{nd} position - cross harp) G major scale. In order to develop a usable (and symmetrical) scale from the registral limitations of both Harmonica and Bass, I first omitted the pitches C5, D5, B5 and C6 from the harp scale (see diagram below).

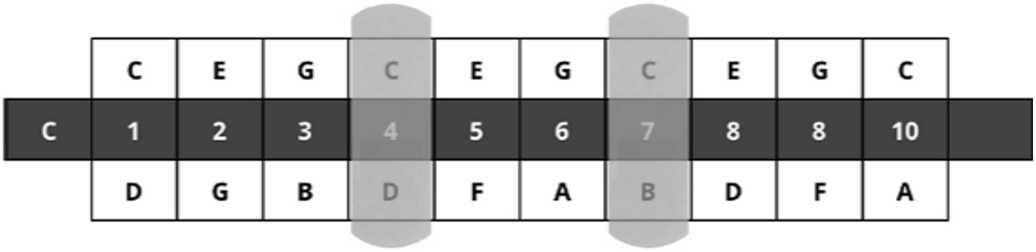


Figure 81: Diatonic "C" Harmonica, limited pitch field

The complete available ascending fifteen-pitch set (from C4) is now {C4, D4, E4, G4, B4, ~~C5~~, ~~D5~~, E5, F5, G5, A5, ~~B5~~, ~~C6~~, D6, E6, F6, G6, A6, C7} = the intervallic set [2, 2, 3, 4, 5, 1, 2, 2, 5, 2, 1, 2, 2, 3]. This scale has two perfect-fourth gaps; B4 to E5 and A5 to D6. The inverse of this set = [3, 2, 2, 1, 2, 5, 2, 2, 1, 5, 4, 3, 2, 2], and when this intervallic pattern is applied to an *abstract* bass range from (e.g.) B0 to B3 (although B0 is of course out of range), the ascending (↑ from B0) bass set= {~~B0~~, D1, E1, F♯1, G1, A1, D2, E2, F♯2, G2, C3, E3, G3, A3, B3}. The complete 30-pitch symmetrical (B0 to C7) LIE scale is now as follows:

Figure 82: 30-pitch LIE scale used in *Biting the Bullet...*

This scale has a (Y2) axial centre at B3/C4, at the top of the bass and bottom of the harp range: {~~B0~~, D1, E1, F♯1, G1, A1, D2, E2, F♯2, G2, C3, E3, G3, A3, **B3** | **C4**, D4, E4, G4, B4, E5, F5, G5, A5, D6, E6, F6, G6, A6, C7}.

As with the trombone part in *An Apology…*, Richard found that the compound intervallic leaps in the opening bars were particularly problematic. This was overcome by placing thin strips of tape (fret-like) across the fingerboard.

In the accompanying video for this piece, the visual juxtaposition of war footage and instrumentalists alludes to my long standing pacifist beliefs. By paraphrasing the Buddhist concept that war would cease if a majority of people were *singing* or *music making* in general, rather than specifically chanting, one arrives at a music specific variant of Shepard Fairey's "Make Art Not War" (Obey Giant, 2019). In this context, the opening and closing LIE scalic derived bass refrain (bb. 1–8 and 89–96) might be thought of as a pacifist refrain (rather than fanfare) for the common man.

String Quartet No. 1 (Where's Noah?) (2024)
Movement No. 1
Duration: 4′ 50″

This movement was initially inspired by the Dover Quartet's precise temporal rendition of Ravel's String Quartet in F Major, Mvt. 2 (see Brooklyn Classical, 2022). The 52-STET Prime scale and various OG sub-scales that form the skeletal scaffolding of this particular movement are derived from the Möbius strip related aspect ratio 1:1.73. A smooth/continuous paper Möbius strip cannot be created from a square - a rectangular shape is required. The aspect ratio of a square is "1" (1:1) and in 2023 Richard Schwartz proved that in order to create a Möbius strip, a usable paper rectangle must have an "aspect ratio greater than √3" (2023, p. 1). The √3 (square root of 3) = 1.73, and the aspect ratio 1:1.73 is here explored as a "limit" using scalic patterns.

Limits are important, but as with the 1.5° of global warming (that was breached in 2023) we *globally* seem to be unable to stick to them. As sea levels and the threat to human life continue to rise exponentially, one might wonder - *Where's Noah*?

If C = “0”, the ascending (↑) Prime (P1) scale = [1, 2, 3, 4, 5] = {C, C♯, E♭, F♯, B♭, E♭}, and the Prime (P1.73) scale (1 x 1.73) = [2, 3.5, 5, 7, 8.5]. Multiply both sets by 2 to create whole numbers, and from a C4 (zero-point) central axis; P1 = [2, 4, 6, 8, 10] = 30-STET scale (A2-E♭5) = {A2, B2, E♭3, A3, F4, E♭5}, and P1.73 = [4, 7, 10, 14, 17] = 52-STET scale (B♭1-D6) = {B♭1, D2, A2, G3, A4, D6}. The Inverse (I) descending (↓) variants are; I1 = [10, 8, 6, 4, 2] = 30-Stet scale (A2-E♭5) = {A2, G3, E♭4, A4, C♯5, E♭5}, and I1.73 = [17, 14, 10 7, 4] = 52-Stet scale (B♭1-D6) = {B♭1, E♭3, F4, E♭5, B♭5, D6}.

The complete centrosymmetric P+I (1 + 1.73) 14-pitch 52-Stet (B♭1-D6) scale becomes {B♭1, D2, A2, B2, E♭3, G3, A3, E♭4, F4, A4, C♯5, E♭5, B♭5, D6} [4, 7, 2, 4, 4, 2, 6, 2, 4, 4, 2, 7, 4].

The complete OG 39-pitch 50-STET scale (B1-C♯6) is therefore:

{B1, C2, C♯2, E♭2, E2, F2, F♯2, G2, G♯2, B♭2, C3, C♯3, D3, E3, F3, F♯3, G♯3, B♭3, B3, C4, C♯4, D4, E4, F♯4, G4, G♯4, B♭4, B4, C5, D5, E5, F5, F♯5, G5, G♯5, A5, B5, C6, C♯6}

[1, 1, 2, 1, 1, 1, 1, 1, 2, 2, 1, 1, 2, 1, 2, 2, 1, 1, 1, 1, 1, 1, 2, 2, 1, 2, 1, 1, 2, 2, 1, 1, 1, 1, 1, 2, 1, 1]

As a compositional device, in this piece the OG scale is divided into 3 sub-scales, as follows; OG1 = 13-pitch 50-STET scale (B1 - C♯6) = [4, 1, 6, 2, 6, 6, 6, 2, 6, 1, 4] = {B1, E♭2, E2, B♭2, C3, F♯3, C4, F♯4, C5, D5, G♯5, A5, C♯6} , OG2 = 14-pitch 48-STET scale (C2 - C6) = [5, 1, 7, 1, 2, 6, 4, 6, 2, 1, 7, 1, 5] = {C2, F2, F♯2, C♯3, D3, E3, B♭3, D4, G♯4, B♭4, B4, F♯5, G5, C6} , OG3 = 12-pitch 46-STET scale (C♯2 - B5) = [6, 1, 9, 3, 3, 2, 3, 3, 9, 1, 6] = {C♯2, G2, G♯2, F3, G♯3, B3, C♯4, E4, G4, E5, F5, B5}. **T**here are now seven possible OG combinatorial permutations; 1, 2, 3, 1+2, 1+3, 2+3, and 1+2+3.

All the compositions in this portfolio utilise various LIE scalic combinations. A visual analogy for my compositional approach might be to compare LIEs with the thick black outlines employed in the painting of (e.g.) Piet Mondrian, Beauford Delaney, and John Bratby (see figure 83). The black outlines are synonymous with the scales, the colour fields relate to the resulting harmony, and the compositional elements are drawn from life. As with 12-tone serial techniques, composing with LIE scales (of course) imposes structural limitations, but as mentioned by Schoenberg, “restrictions imposed on a composer… can only be overcome by an imagination which has survived a tremendous number of adventures” (1950, p. 114).

Figure 83: *Chalking the Cue*: Oil on canvas (Bratby and Clutton-Brock, 1961, Plate 14)

Conclusion

> The bass moves into the middle: this is our musical revolution. Several composers after Webern, myself included, have been fascinated by harmonic structures which radiate out from either side of a central axis in reflecting intervals. Unless a strong contrary line is taken in atonal music the bass will remain at the bottom of what sounds like dissonant music. But in symmetrical mirroring structures it is forced, focal attention is forced, into the axial middle, because all relationships converge there: the sounds point to it (Jonathan Harvey, 1982, p. 2)[84]

Much post-diatonic experimentation might be defined by the negation, incorporation, avoidance or exclusion of Rameau's (1722) concept of "basso continuo", or fundamental bass (Thomson, 1993). To effectively avoid what Schoenberg refers to as the "somnambulistic sense of security" (1950, p. 102) which is regulated through "root progressions" (ibid.), [85] one must embrace the idea that "whatever sounds simultaneously is a harmony" (Ziehn, 1907, p. 3). Similarly, "post-tonal" harmonic *prolongation* (e.g. Travis, 1966; Lerdahl, 1997; Harper, 2007) through extended linear intervallic "association" (Straus, 1987) is here (generally) assumed. As a rock drummer who has been lucky enough (on many occasions) to perform with some exceptional bass players, [86] compositionally re-purposing the concept of bass *roots* remains a particular stumbling block on the way to elucidating any alternative harmonic verities, and LIE scales are (to a certain extent) a means to an end in

[84] In this article, Harvey also suggests that a musical shift "away from frenetic teleologies and obsessive dynamism... may reflect the spirit of the ending of the late capitalist phase of the West" (1982, p. 4). I tend to think that *embracing* the frenetic might be more musically applicable, and remember debating this with Jonathan on various occasions. Either way, Harvey concludes that "it is legitimate... to speak of revolutions, at least in the domains of pitch and timbre" (ibid.).

[85] Octaviated structures clearly exert a perceptual (Do-to-Do) and numerical (0 = 12 = 24...) bias towards the fundamental (zero/starting) pitch. Mathematically, zero is however included in the set of natural numbers "whenever convenient" (Ribenboim, 1996, p. 530).

[86] This includes: Matt Jury (*House of Love*), Richard Itchington (*999, Bram Tchaikovsky, Sussex*), Ricky McGuire (*UK Subs*, *TMTCH*), Steve Dean (*Surfin' Lungs*), and Terry Mann (*Taj Mahal).*

this particular regard. Perhaps all mathemusical experiments using consecutive counting numbers will eventually lead back to the Pythagorean tetractys (Figure 82), which geometrically defines the relationship between the first four triangular numbers (1, 3, 6, 10) and the consecutive counting numbers 1: 2: 3: 4.

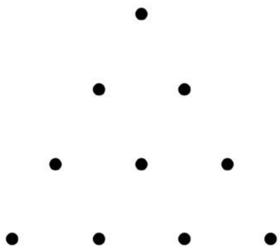


Figure 84: 10-point *tetractys*

When rendered as a 10-STET LIE scale, (if, e.g. C4 and B♭4 are the vertices) the ascending pitch scale of the tetractys = {C [1] C♯ [2] E♭ [3] F♯ [4] B♭}, the descending scale = {B♭ [1] A [2] G [3] E [4] C}, and the combinatorial P+I scale = {C [1] C♯ [2] E♭ [1] E [2] F♯ [1] G [2] A [1] B♭} (the pitches of Messiaen's Mode 2). However, when arpeggiated and inverted, the descending scale produces the basic Jazz/blues "walking" bass line {0, 4, 7, 9, 10, 9, 7, 4, 0} (see Figure 83) that adds the major sixth and minor seventh to the major triad, and it is this that I have attempted to compositionally avoid and/or deconstruct.

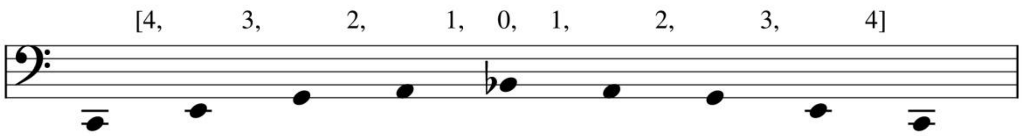


Figure 85: "Walking" bass line

Throughout this research (2017-2024) I have compositionally explored the aesthetic potential of centrosymmetric LIE Scales, and the process itself has provided me with an idiosyncratic generative vehicle for achieving my ultimate aim of writing large-scale acoustic pieces (see Appendix III). I hope that this LIE scalic endeavour (from theory to outcomes) demonstrates that (as with e.g., fractals and the Mandelbrot set) "profound intricacy can emerge from the simplest of rules" (Cepelewicz, 2024). In this thesis I have collated and analysed numerous LIE scalic structures, and although elucidating any potential through composition is what matters, one must also be mindful of what Berio calls "imitation grammar", in that

> any attempt to codify musical reality into a kind of imitation grammar... is a brand of fetishism which shares with Fascism and racism the tendency to reduce live processes to immobile, labelled objects, the tendency to deal with formalities rather than substance (Berio, 1996, p. 169).

On the other hand, "…mathematics enables us to perceive curious analogies between music and phenomena (apparently) very far removed from it… and suggests to us analogies which… might have remained unnoticed" (Combarieu, 1910, p. 293). In this respect, although I believe that a more interdisciplinary approach might "challenge our cognitive capacity by suggesting new points of reference and of departure" (Vanmaele, 2017, p. 285), this is generally beyond the scope of this thesis, and has only been alluded to in passing.

Correspondences between pitches and integers have been employed and/or inferred since writing on music began, [87] but John Rahn's cautionary words are worth reiterating. "We must carefully determine the limits of similarity between integers (with their structure) and pitches (with their possible structures), to do otherwise would be to fall into *numerological fallacy*" (Rahn, 1980, p. 19). However, Tsao points out, that "one of the challenges of

[87] See Andreatta (2015) for an historical overview of the interplay between music and maths.

contemporary music composition is to speculate upon *possible worlds*" (2017, p. 44), and if we are to *spark* the noetic imagination, then discussing music in *mathemusical* terms is one of many valid approaches that might be taken. Not forgetting that "math is not music, and one must remember to distinguish between a task and the tools used to accomplish it" (Morris, 2007, p. 99).

Peter Toth suggests that "symmetrical pitch constructions lay at the heart of the transformation of the traditional tonal system" (2016, p.151), and Ives reminds us that the "right hand up… left hand down..." (1973, p. 42) chords derived from his "piano-drum-writing" (ibid.) technique (mentioned in the Preface of this thesis) and "piano cycle rhythm studies - 2-3-5-7-11-7-5-3-2- etc…" (1973, p. 36) are evident in numerous of his pieces.[88] However, to my knowledge, no composer has specifically used compound centrosymmetric (P+I) scales with STET limits and OG elements as a generative source. Having immersed myself in this LIE Scalic sound world for almost a decade, I tend to concur with Ives's (following) insightful statement.

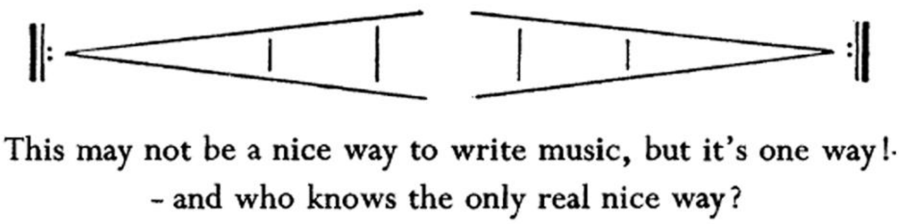


Figure 86: Symmetrical sketch from *Memos* (Ives, 1973, p. 64)

[88] "1776, Country Band March, In The Inn, Calcium Light Night, Over The Pavements, Second Violin Sonata, Hawthorn, The Fourth of July, Putnam's Camp, General Booth, Fourth Symphony" (Ives, 1973, p. 42). Many Western musical paths/concepts lead to back to Ives, to the extent that the term *atonal* could perhaps be referred to as *itonal*.

LIE scales are an ongoing source of inspiration and revelation. Only when compiling the scale (derived from the Fibonacci sequence) used for my *Symphony No. 9* (2024, see Appendix III), did I find that the IC cardinality of the OG scale contains exactly 7 minor thirds [3], 14 whole tones [2], and 28 semitones [1] (see diagram below). If “solving a simple arithmetic problem seems to be less complex and to require fewer and less extended cerebral networks than when listening to music” (Petsche *et al*., 1993, p. 129), then mathematicians might be able to explain why this parsimonious doubling pattern emerges? Musically, this is just one example of why LIE scales are (as demonstrated throughout this thesis) a potentially rich generative vein for creative endeavour.

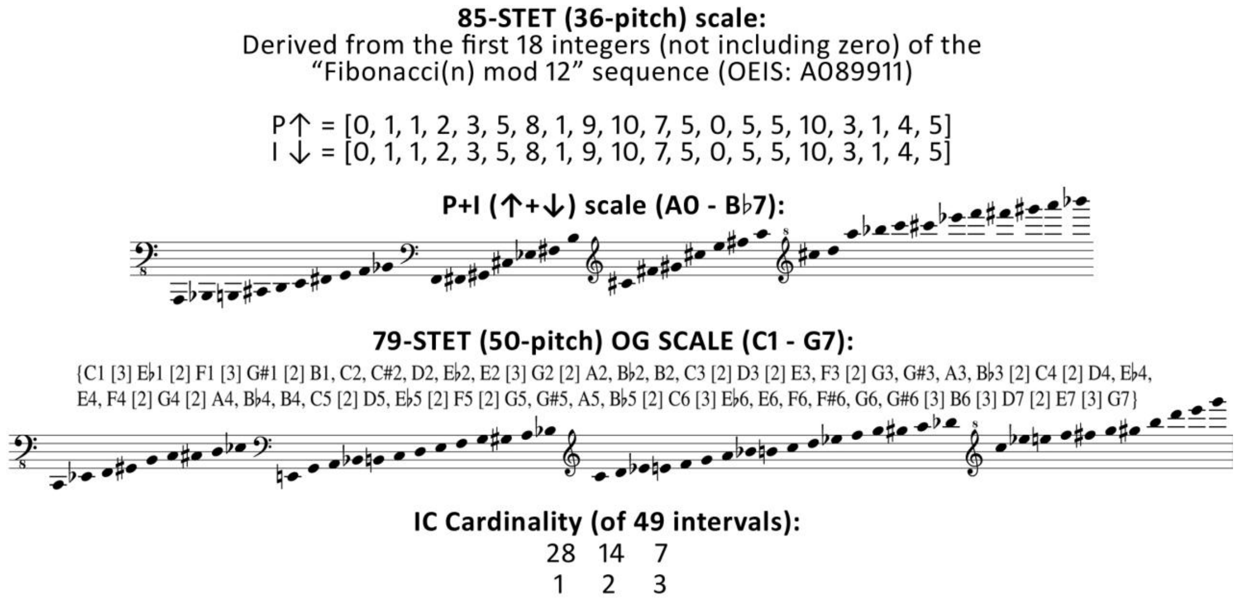
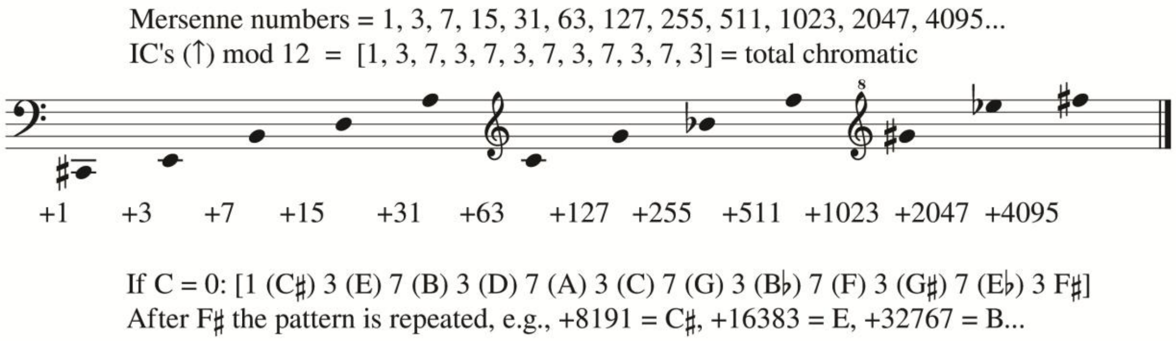


Figure 87: Scales derived from the “Fibonacci… mod 12” sequence

Appendix I – Some remarkable LIE scalic pitch-integer correspondences

Leibniz's use of Mersenne Numbers and an unfolding chromatic

In Leibniz's first publication, *Dissertation on the Art of Combinations, 1666* (1989, p. 7), Leibniz uses the first twelve integers of the sequence known as the "Mersenne numbers" 1, 3, 7, 15, 31… (OEIS: A000225).[89] Why only twelve? Leibniz suggests that "this is far enough, since it is easily extended" (1989, p. 79), and when the first twelve Mersenne *prime* numbers are rendered as a cumulative unidirectional 'P' scale, the result produced is an exact unfolding of the twelve pitches of the chromatic scale (see diagram below). This sequence might also be thought of as a *step scale* (up 3, down 5); {C♯ [↑3] E [↓5] B [↑3] D [↓5] A [↑3] C [↓5] G [↑3] B♭ [↓5] F [↑3] G♯ [↓5] E♭ [↑3] F♯ [↓5] C♯…}.



As Leibniz was a theorist with an awareness of Aristoxenus's additive (ladder model) interval calculus (Bühler, 2010) pertaining to a "twelve equal division system" (Bühler, 2019, p. 44), one might surmise that Leibniz was aware of this particular extended interval/integer correspondence.[90] In the diagram below, the annotated version of Leibniz's original table (related to the complexions of exponents) should help to explain this.

The Mersenne numbers are equivalent to the powers of 2 minus 1 (2^n-1), and all the numbers in Leibniz's table are derived from simply adding zero to the first integer in column 1 (row 1) and placing the result in column 2, row 2 (e.g. 1 + 0 = 1, 2 + 1 = 3, 3 + 3 = 6, 20 + 15 = 35 etc). From this binary starting point we are thus taken to the triangular numbers (row 2), tetrahedral numbers (row 3), and binomial coefficient sequences from row 4 onwards. From this additive process intrinsic centrosymmetric patterns emerge (in the diagram below

[89] In his *Explanation of Binary Arithmetic* (1703) Leibniz also uses (and underlines) the Mersenne sequence to draw attention to the repetitive cyclical patterns found within the binary system (1703, p. 87), in that the Mersenne Numbers correspond to 1_2, 11_2, 111_2, 1111_2, 11111_2… when written in binary.

[90] If so, this might have appealed to his game-theoretic reductionist panpsychism. In "*On the ultimate origination of things*" (1697), Leibniz states that "out of the infinite combinations of possibilities, and possible series, there exists one through which the greatest amount of essence or possibility is brought into existence. There is always in things a principle of determination which must be sought in maximum and minimum; namely, that the greatest effect should be produced with the least expenditure" (1697).

these are highlighted in grey), culminating in an exact representation of the 4,095 possible 12-tone scalic permutations.

Discussing the pitched (which Leibniz refers to as "registral") attributes of wind organs, Leibniz asks us to "assume that there are in some organs only twelve such simple effects; then there will be in all about 4095" (1989, p. 81).

(12 + 66 + 220 + 495 + 792 + 924 + 792 + 495 + 220 + 66 + 12 + 1 = 4095) **No. of 12-tone scalic permutations**

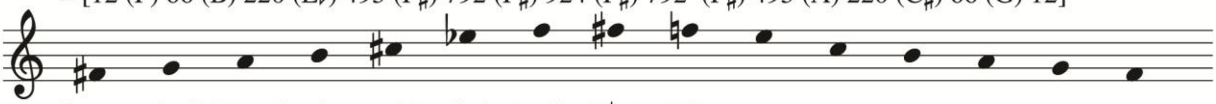
1		1	2	3	4	5	6	7	8	9	10	11	12	= Monotonic (1)
2		0	1	3	6	10	15	21	28	36	45	55	66	= Ditonic (2)
3			0	1	4	10	20	35	56	84	120	165	220	= Tritonic (3)
4				0	1	5	15	35	70	126	210	330	495	= Tetratonic (4)
5					0	1	6	21	56	126	252	462	792	= Pentatonic (5)
6						0	1	7	28	84	212	462	924	= Hexatonic (6)
7							0	1	8	36	120	330	792	= Heptatonic (7)
8								0	1	9	45	165	495	= Octatonic (8)
9									0	1	10	55	220	= Nonatonic (9)
10										0	1	11	66	= Decatonic (10)
11											0	1	12	= Hendecatonic (11)
12												0	1	= Chromatic (12)
*	0	1.	3.	7.	15.	31.	63.	127.	255.	511.	1023.	2047.	4095.	= TOTAL

Adapted from Leibniz, 1989, p. 79; originally (1666, p. 7)

Leibniz's "*N*" table, column 12 (1989, p. 17)

When rendered as a unidirectional LIE scale, this sequence corresponds to the Neapolitan Major (the 11th Mela Raga) scale.

From an axial "C" [12, 66, 220, 495, 792, 924, 792, 495, 220, 66, 12]
= [12 (F) 66 (B) 220 (E♭) 495 (F♯) 792 (F♯) 924 (F♯) 792 (F♯) 495 (A) 220 (C♯) 66 (G) 12]



Symmetrical OE reduction = {F♯, G, A, B, C♯, E♭, F, F♯}
= [1, 2, 2, 2, 2, 2, 1] = Neapolitan Major (11th Melakarta Raga) Scale

Kaprekar numbers; another example of Messiaen's "mode 2" congruence

Kaprekar numbers (not to be confused with the Kaprekar routine or constant, which produces the number 6174) are any *n*-digit number which, when squared, the right and left digits add up to the original n-digit number. For example, "9 is a Kaprekar number since 9^2 = 81, and 8 + 1 = 9. The number 297 is also a Kaprekar number since 297^2 = 88209, and 88 + 209 = 297" (Weisstein, 2003).

The first 30 Kaprekar numbers (OEIS: A006886) are; 1, 9, 45, 55, 99, 297, 703, 999, 2223, 2728, 4879, 4950, 5050, 5292, 7272, 7777, 9999, 17344, 22222, 38962, 77778, 82656, 95121, 99999, 142857, 148149, 181819, 187110, 208495, 318682.

The mod 12 PC values for these Kaprekar numbers (if "C" = zero) are: C♯, A, A, G, E♭, A, G, E♭, E♭, E, G, F♯, B♭, C, C, C♯, E♭, E, B♭, B♭, F♯, C, A, E♭, A, A, G, F♯, A, B♭. Remarkably, if 'C' = 0, 12, 24..., these PCs are all unique to Messiaen's mode 2 {C, C♯, E♭, E, F♯, G, A, B♭}; in other words, no Kaprekar number corresponds to the (OG) pitches of the Diminished Seventh {D, F, G♯, B} {2, 5, 8, 11}.

The Bell numbers (OEIS: A000110)
Mod 12 = (1, 1, 2, 5, 3, 4, 11, 1, 0, 3, 7, 6, 1, 1, 10, 5, 11, 0, 7, 5, 5, 3, 7, 8, 0, 5, 1)
Integers belonging to this sequence = {0, 1, 2, 3, 4, 5, 6, 7, 8, 10, 11}, which (if 'C' is zero)
= the complete chromatic minus the relative minor, 'A' {9}.

Tchoukaillon Solitaire Sequence

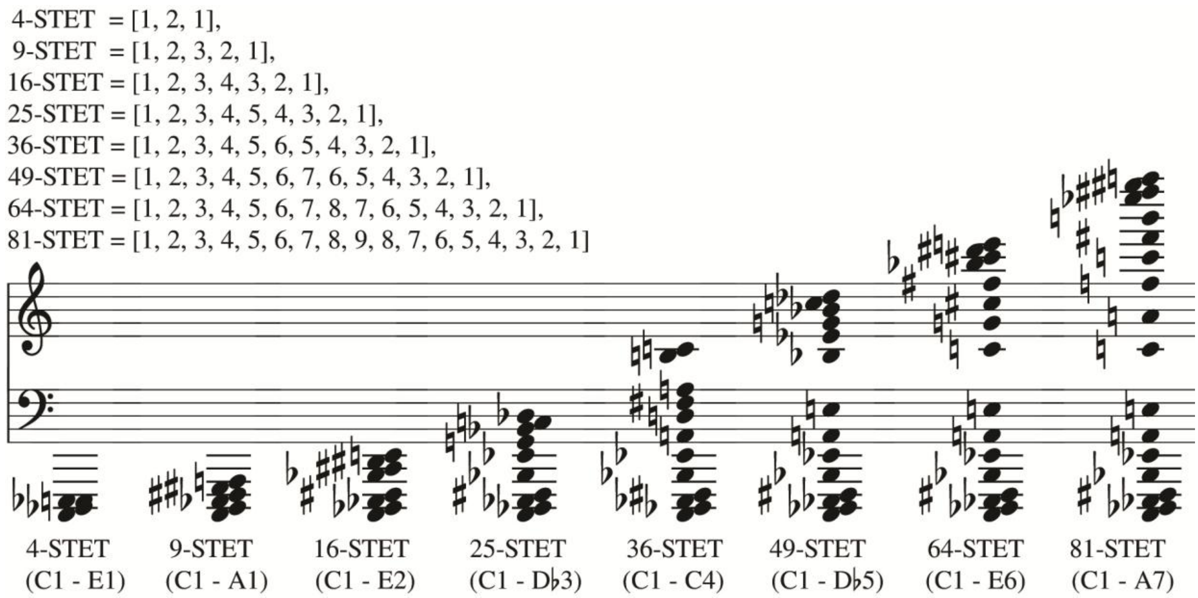
The integer sequence that defines the "Strings giving winning positions in Tchoukaillon (or Mancala) solitaire" (OEIS: A028931) is as follows: 0, 1, 20, 21, 310, 311, 4200, 4201, 4220, 4221, 53110, 53111, 642000, 642001, 642020, 642021, 642310, 642311, 7531200, 7531201, 7531220, 7531221, 86420110, 86420111, 86424000, 86424001, 86424020, 86424021, 86424310, 86424311. If, ascending, C = 0, G♯ = 20, B♭ = 310..., only the PCs of the hexatonic scale {C, C♯, G♯, A, B♭, B} are produced. The OE semitonal progression from G♯ ↑ C♯ spans a 5-STET perfect fourth [5] and avoids the tritone [6] in both directions.

Centrosymmetric odd numbers and the square number sequence

Although we define certain sequences as linear, and others as exponential or "power" sequences, simply adding the linear sequence of odd numbers (0, 0+1, 1+3, 1+3+5, 1+3+5+7...) produces the exponential *square number* sequence (0, 1, 4, 9, 16, 25..., OEIS: A000290), which cumulatively (using semitones) produces the unique (if 'C' = 0) tetrachord {C, C♯, E, A} {0, 1, 4, 9}. This can be expressed as a [3, 1, 3] tetrachord ascending from 'A' {A, C, C♯, E} that combines both the major [4, 3] and minor [3, 4] triads.[91] Using a centrosymmetric structural run of counting numbers, i.e. **1**, 1+**2**+1, 1+2+**3**+2+1,

[91] For major [4] and minor [3] extensions of George Russell's Lydian chord, see Appendix II of this thesis.

1+2+3+**4**+3+2+1..., also results in STETs of 1, 4, 9, 16, 25..., but produces *alternative* chordal/scalic permutations.



13 positive integers and the "Magen Abot II" scale

There are exactly (and only) 13 positive integers that are not the sum of 4 hexagonal numbers (OEIS: A007527), these are: 5, 10, 11, 20, 25, 26, 38, 39, 54, 65, 70, 114, and 130. The 12 intervallic distances between the integer values of the above sequence are: |5|,|1|,|9|,|5|,|1|,|12|,|1|,|15|,|11|,|5|,|44|,|16|.

Using ascending LIE cumulative expansion (from 'C'), the above sequence corresponds to the PCs; (C), F, F♯, E♭, G♯, A, A, B♭, C♯, C, F, C♯, F = {C, C♯, E♭, F, F♯, G♯, A, B♭} {0, 1, 3, 5, 6, 8, 9, 10} in ascending OE order. This particular integer sequence/scale is unique to the sum of first *n* terms of the Thue-Morse sequence (OEIS: A026430). Ascending from 'A', the IC sequence is [1, 2, 1, 2, 2, 1, 2, 1], a recursive isomorphic pattern known as the "Magen Abot II" scale.

Squares of primes (OEIS: A001248/list)

4, 9, 25, 49, 121, 169, 289, 361, 529, 841, 961, 1369, 1681, 1849, 2209, 2809, 3481, 3721, 4489, 5041, 5329, 6241, 6889, 7921, 9409, 10201, 10609, 11449, 11881, 12769, 16129, 17161, 18769, 19321, 22201, 22801, 24649, 26569, 27889, 29929, 32041, 32761, 36481, 37249, 38809, 39601, 44521, 49729, 51529, 52441, 54289, 57121, 58081, 63001, 66049, 69169, 72361, 73441, 76729, 78961, 80089, 85849, 94249, 96721, 97969, 100489, 109561, 113569, 120409…

When rendered as cumulative LIE intervals (from 'C'↑), +4 = E, +9 = C♯, +25 = D, etc. After the initial *n*1 pitch 'E' (C+4), the following ascending array is produced: {C♯, D, E♭, E, F, F♯, G, G♯, A, B♭, B, C, C♯, D…}. In other words, from *n*2 (9) onwards, this sequence corresponds *exactly* to the ascending chromatic scale.

Pitches related to the list of twin primes (OEIS: A077800)

As an ascending cumulative LIE sequence, if C= 0, 12, 24..., then {+3+5+5+7+11+13+17...} = {E♭, G♯, C♯, G♯, G, G♯, C♯...}. After the opening E♭ (C+3), the only corresponding PCs utilised in A077800 are G♯, C♯, and G (see list below*),* and *all* occurrences of G's and C♯'s are nested between G♯'s. Similarly, after the opening pair, all the sums of the twin prime pairs (see OEIS: A054735) are multiples of 12.

The first 44 twin prime pairs:
(3, 5), (5, 7), (11, 13), (17, 19), (29, 31), (41, 43), (59, 61), (71, 73), (101, 103), (107, 109), (137, 139) (149, 151) (179, 181), (191, 193) ,(197, 199), (227, 229), (239, 241), (269, 271), (281, 283), (311, 313), (347, 349), (419, 421), (431,433), (461, 463), (521, 523), (569, 571), (599, 601), (617,619), (641, 643), (659, 661), (809, 811), (821, 823), (827, 829), (857, 859), (881, 883), (1019, 1021), (1031,1033), (1049,1051), (1061,1063), (1091, 1093), (1151, 1153), (1229, 1231), (1277, 1279), (1289, 1291).

Resulting PCs from the above list:
E♭, G♯, C♯, G♯, G, G♯, C♯, G♯, C♯, G♯, C♯, G♯, G, G♯, G, G♯, C♯, G♯, G, G♯, C♯, G♯, C♯, G♯, G, G♯, G, G♯, C♯, G♯, G, G♯, G, G♯, C♯, G♯, C♯, G♯, G, G♯, G, G♯, G, G♯, G, G♯, C♯, G♯, C♯, G♯, C♯, G♯, G, G♯, C♯, G♯, C♯, G♯, G, G♯, C♯, G♯, C♯, G♯ G, G♯, C♯, G♯, C♯, G♯, G, G♯, G, G♯, C♯, G♯,C♯, G♯, G, G♯, G, G♯, C♯, G♯, C♯, G♯, C♯, G♯.

Digits of Pi in relation to consecutive counting numbers

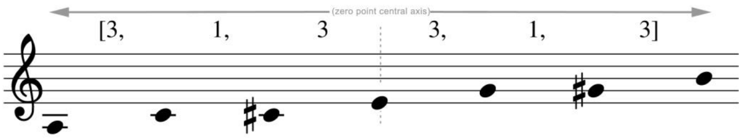
The occurrence of centrosymmetric (axiomatic) triplets within the "Decimal expansion of Pi" (OEIS: A000796) is as follows; **1**, 4, **1** - **5**, 3, **5** - **9**, 7, **9** - **3**, 2, **3** - **6**, 2, **6**, - **3**, 8, **3** - **9**, 3, **9** - **4**, 9, **4**.... The outer edges (truncated vertices) of the first six triplet sets are [1, 5, 9, 3, 6, 3] (this sequence is unique to OEIS: A116191).[92] When rendered as a cumulative (↑) LIE scale (if 'C' = zero), [1, 5, 9, 3, 6, 3] produces the ordered pitch sequence {C♯, F♯, E♭, F♯, C, E♭}, the OE (PC set) variant of which is {C, C♯, E♭, F♯} or [+1, +2, +3], the counting numbers.

[92] Perhaps this relates to Zilber's *structure* classes, leading to the conjecture that "the unique model of cardinality continuum is isomorphic to the field of complex numbers with exponentiation" (Zilber, 2005, p. 67).

Appendix II – Repository of LIE scales >12 (indexed by STET)

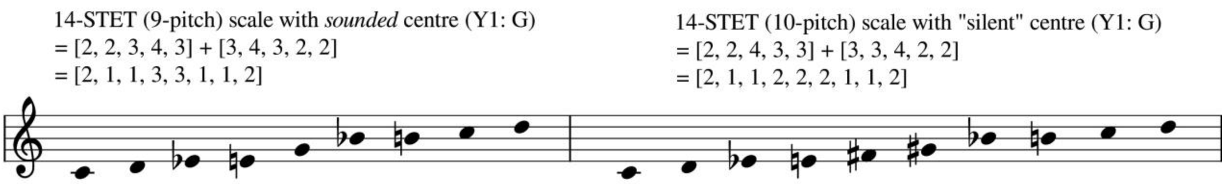
14-STET heptatonic bidirectional variant of Hauptman's [3, 4 | 4, 3] scale

[3, 4 | 4, 3] + [4, 3 | 3, 4] = [3, 1, 3 | 3, 1, 3] {A, C, C♯, E, G, G♯, B}



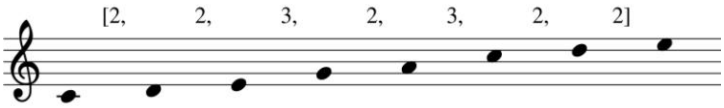
(MP3 here)

14-STET (9-pitch and 10-pitch) scales



(MP3 here)

16-STET (8-pitch) *major* pentatonic extension [2, 2, 3, 2, 3, 2, 2]

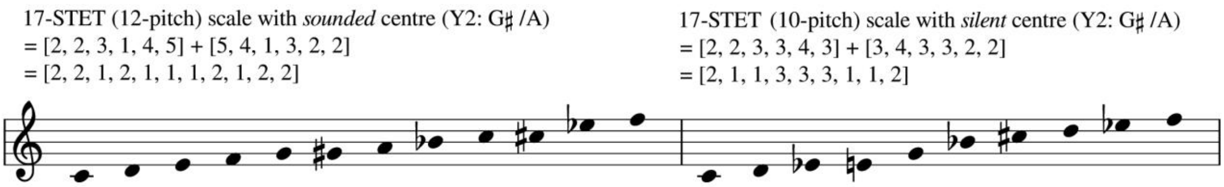


(MP3 here)

17-STET (8-pitch) *minor* pentatonic extension [3, 2, 2, 3, 2, 2, 3]

(MP3 here)

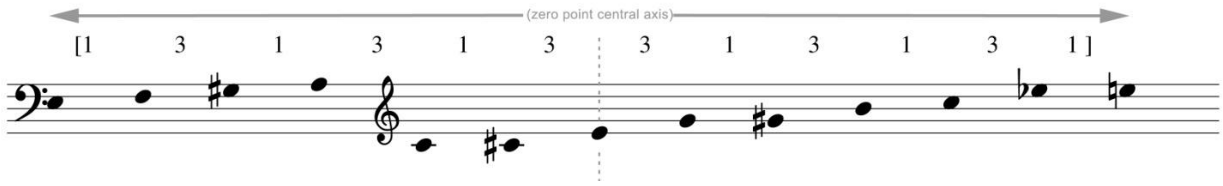
17-STET (12-pitch and 10-pitch) scales



(MP3 here)

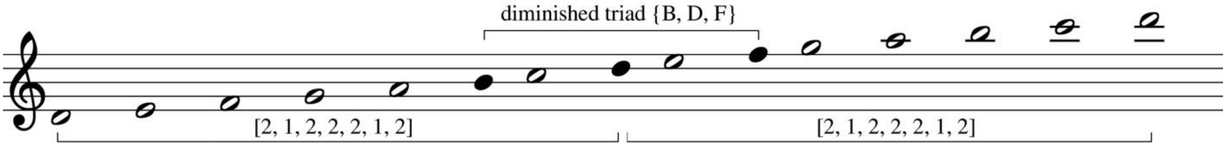
24-STET (13 pitch) extension of the 14-STET [3, 1, 3 | 3, 1, 3] Hauptman scale

The PCs of this scale [1, 3, 1, 3, 1, 3 | 3, 1, 3, 1, 3, 1] correspond to the nonatonic Genus Chromaticum scale (aka Messiaen's Mode 3 and/or Tcherepnin's Augmented Ninth).



(MP3 here)

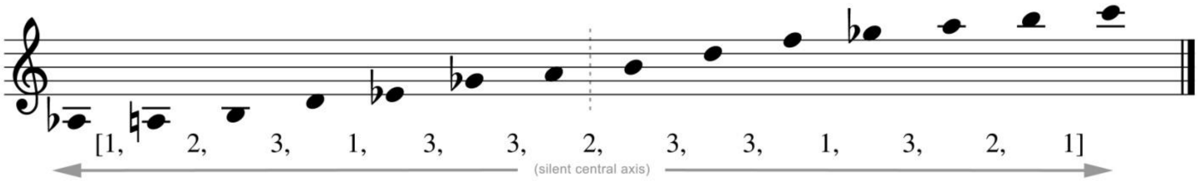
24-STET (15 pitch) Double Dorian systēma



Double (24-STET) Dorian systēma

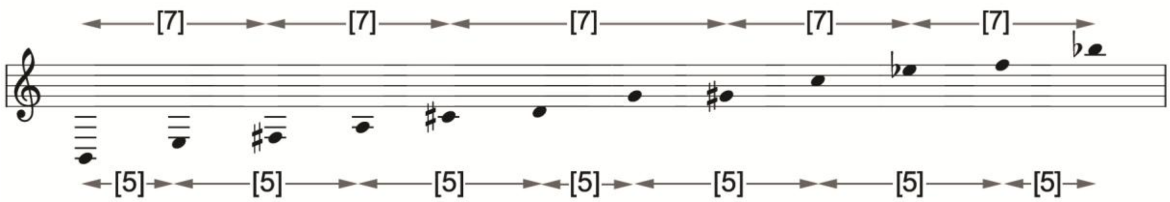
(MP3 here)

28-STET (14-pitch) P+I scale; used in *Fanfaronade* (2022)



(MP3 here)

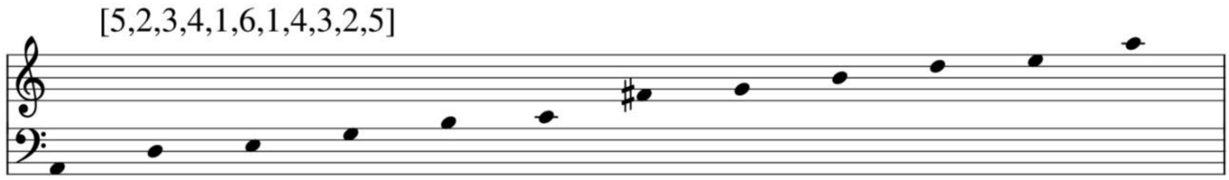
35-STET "fifth duality" scale that produces the complete chromatic



(MP3 here)

36-STET extension of Carey and Clampitt's "normal form of interval class" (1989, pp. 192–193)

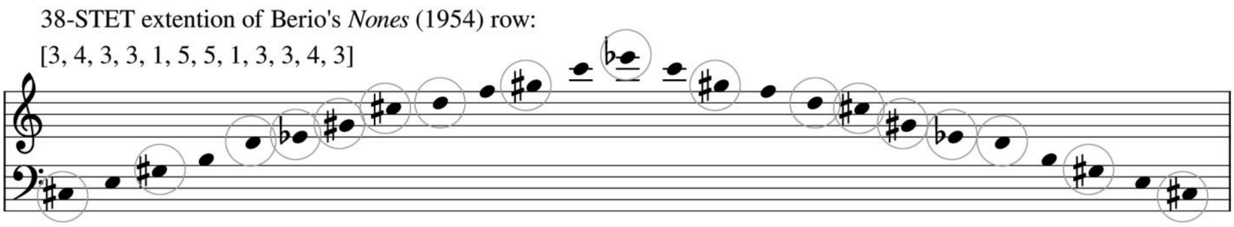
The sequence [5,2,3,4,1,6,1,4,3,2,5] also appears in relation to Kempner (A.K.A Smarandache) Numbers (OEIS: A002034), and the "minimal number of rectangles with integer sides that will form any rectangle with area n" (OEIS: A034880).



(MP3 here)

38-STET scale derived from Berio's 13 note tone row

A registrally and intervallicaly extended LIE scalic version of Berio's *Nones* (1954) series might be the following 38-STET collection: C♯3 [3] E3 [4] G♯3 [3] B3 [3] D4 [1] E♭4 [5] G♯4 [5] C♯5 [1] D5 [3] F5 [3] G♯5 [4] C6 [3] E♭6.



(MP3 here)

38-STET (21-pitch) scale

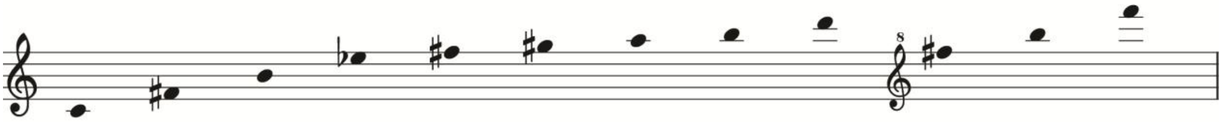
A P+I extension of the [1, 1, 1, 1, 5, 1, 7, 2, 3, 5, 11] scale (see Figure 5, p. 20 and p. 75 of this thesis). This scale is explored in *Étude No.4* (2018).

(MP3 here)

39-STET (30-pitch) OG scale (descending entry order); used in *A gilded cage* (2017)

(MP3 here)

41-STET (12-pitch) P+I scale (ascending entry order); used in *A gilded cage* (2017)



(MP3 here)

42-STET (13-pitch) scale

The scale implied by Vincent D'Indy's "Table of… Tonal Functions" (Montgomery, 1946, p. 172), and used in the construction of a 66-STET prime scale - *Vincent and the Maverick Sonority* (2021).

(MP3 here)

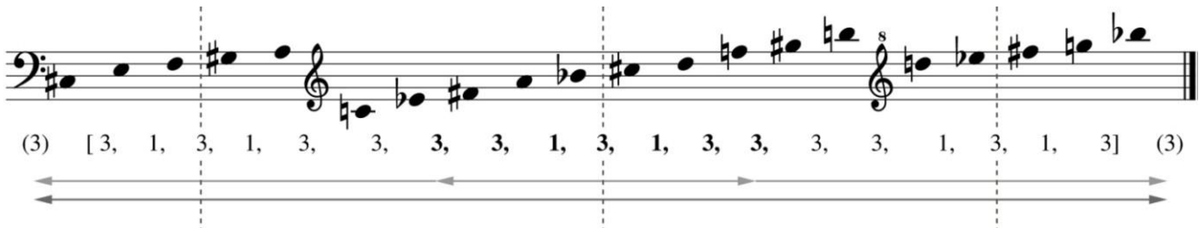
45-STET (10-pitch) scale, after Ives

A LIE scalic rendering of Ives's "2-3-5-7-11-7-5-3-2-... studies in space, time, duration, accent, pulse" (Ives, 1973, p. 36).

(MP3 here)

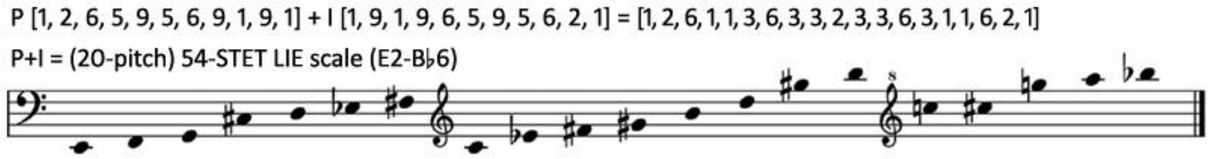
45-STET (20-pitch) scale; used in *3313133* (2021)

This scale has silent double (Y2) axial points at F♯3/G3 - B4/C5 - E6/F6.



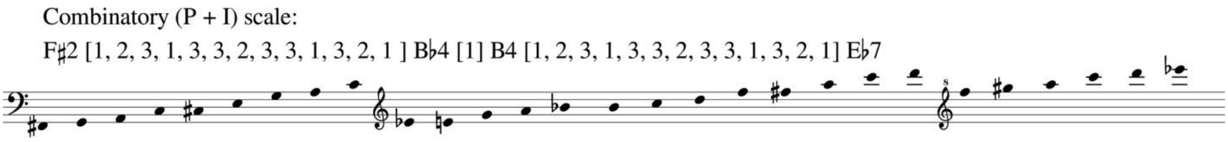
(MP3 here)

54-STET P+I scale; derived from the prime row used in Schoenberg's *Suite for Piano* (Op. 25)



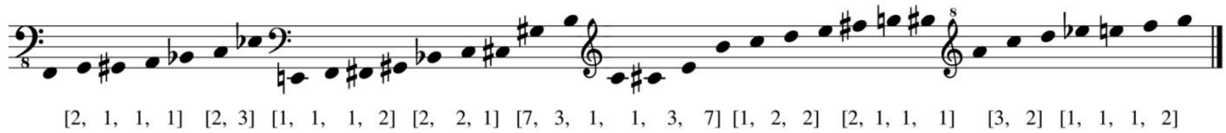
(MP3 here)

57-STET scale
The 28-pitch (P+I) scale (F♯2 ↑↓ E♭7) utilised in *28 Occasions of Experience* (2017). This P+I IC sequence is unique to the “Concatenation of subsequences...” (OEIS: A136436).



(MP3 here)

62-STET (33-pitch) scale; used in *[1, 4] cells* (2021)



(MP3 here)

62-STET (34-pitch) OG scale; used in *Structural Declaration No.1* (2021)

(MP3 here)

66-STET (16-pitch) adaptation of Murail's 'C' dynamics in "Spectra and sprites" (2005)

Murail suggests that "non-octave space [is] arbitrary" (2005, p. 138), primarily as a means of conceptually validating a fused harmonic-timbral continuum not reliant on dodecaphonic (or any other countable) divisions. Murail's polemical article "Spectra and sprites" (2005)[93] might be the last place one would expect to find correlations with a 66-STET ET scale. However, this is what Murail's spectral analysis of C_1 piano tone dynamics (a principle behind his *Désintégration* realised at Ircam in 1982–83) produces. In Murail's dynamic analysis (reproduced below), the first ten loudest partials in descending order of amplitude are 7, 11, 13, 12, 21, 5, 6, 4, 29, 10. To the nearest ET semitone (using Norris's Harmonic Series Calculator, 2015), this list correlates with the pitches B♭, F♯, A♭, G, F, E, G, C, B♭, and E.

2.	1.000000	22.	0.187619	42.	0.119517
3.	0.263176	23.	0.314130	43.	0.120805
4.	0.501411	24.	0.016412	44.	0.026597
5.	0.544941	25.	0.048377	45.	0.050187
6.	0.543653	26.	0.053830	46.	0.019840
7.	0.964906	27.	0.345389	47.	0.029756
8.	0.004356	28.	0.340021	48.	0.006626
9.	0.234125	29.	0.483649	49.	0.010768
10.	0.410792	30.	0.285539	50.	0.024480
11.	0.869808	31.	0.052427		
12.	0.702620	32.	0.006994		
13.	0.703479	33.	0.056200		
14.	0.030799	34.	0.081938		
15.	0.275385	35.	0.168016		
16.	0.009540	36.	0.112062		
17.	0.239186	37.	0.196270		
18.	0.194920	38.	0.100190		
19.	0.394687	39.	0.043469		
20.	0.260476	40.	0.013191		
21.	0.690779	41.	0.031904		

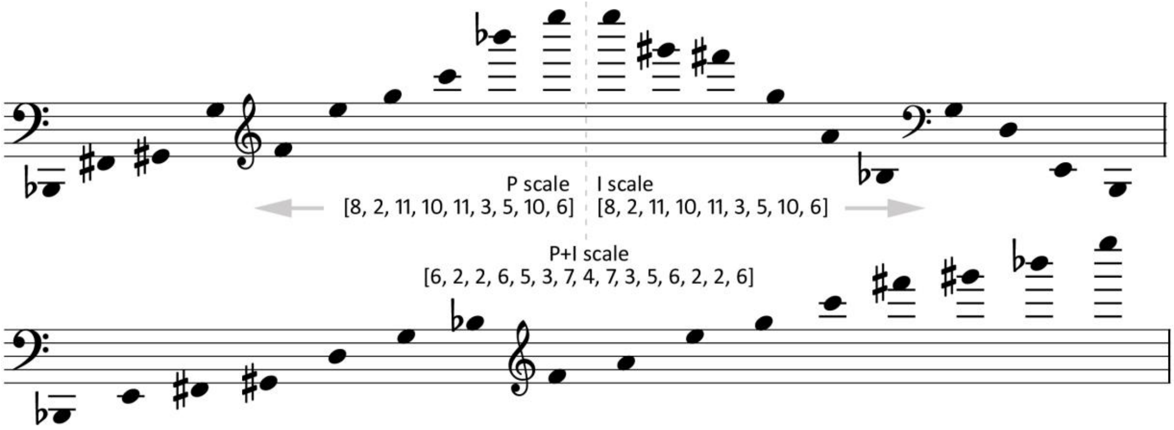
(Reproduced from Murail, 2005, p. 140)

When rendered as a cumulative ascending LIE scale, the intervallic distance between these pitches adds up to 66 (see table below).

Loudest Partials #	7	11	13	12	21	5	6	4	29	10	
Amplitude	0.965	0.869	0.7034	0.7026	0.691	0.5449	0.544	0.501	0.4836	0.4107	
Closest ET pitch	B♭	F♯	A♭	G	F	E	G	C	B♭	E	
Intervallic distance from previous pitch	0	+8	+2	+11	+10	+11	+3	+5	+10	+6	= 66

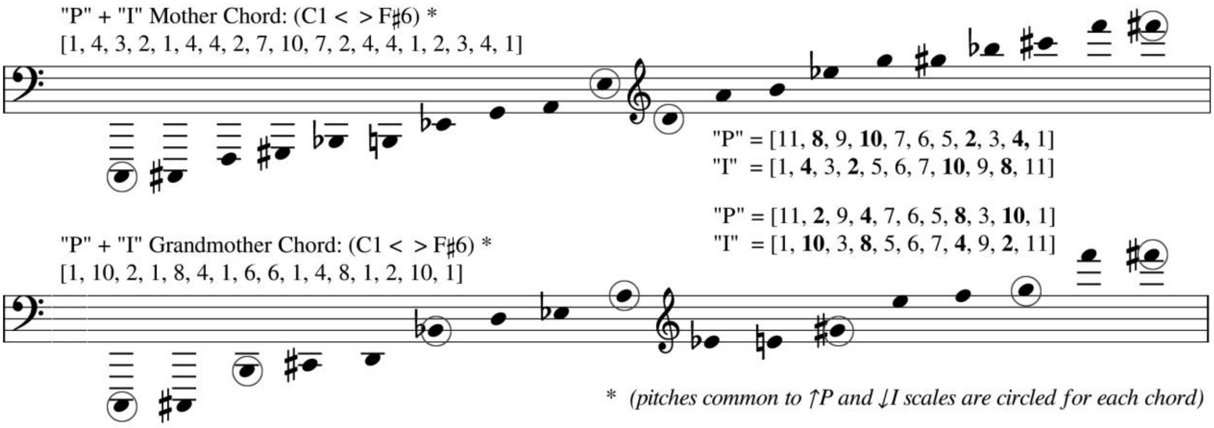
[93] This article first appeared in the Contemporary Music Review under the title "Spectra and pixies" (1984).

Taking a 66-STET span from B♭1 to E7, the P+I combinatorial LIE scale produced is as follows:



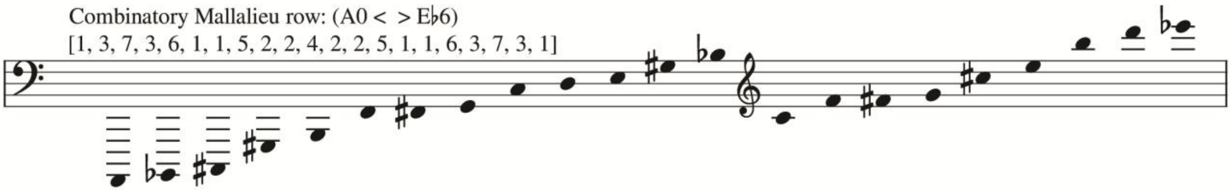
(MP3 here)

66-STET "all interval" scales derived from the "Mother" and "Grandmother" chords



(Mother MP3 here, Grandmother MP3 here)

66-STET (22-pitch) P+I extensions of the "Mallalieu" row



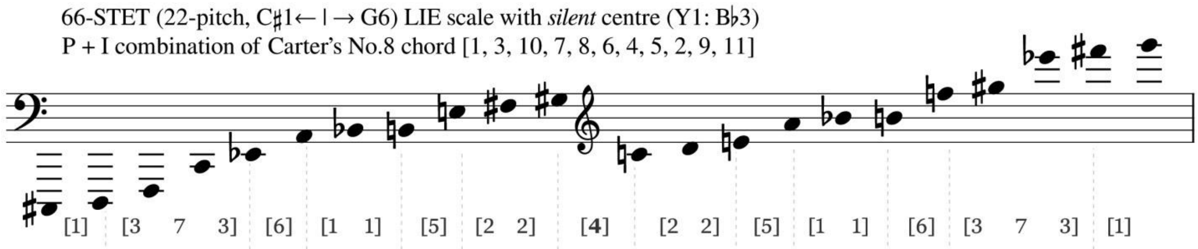
(MP3 here)

66-STET (22-pitch) P+I scale derived from Bauer-Mengelberg and Ferentz

Bauer-Mengelberg and Ferentz discuss the inverted scale of the "permutation 3, 7, T, 1, 5, 2, 9, 4, 6, 8, E" (1965, p. 103), but fail to combine (P+I) this *all interval* scale using common vertices. In short, [3, 7, 10, 1, 5, 2, 9, 4, 6, 8, 11] + [11, 8, 6, 4, 9, 2, 5, 1, 10, 7, 3] produces the following scale with a silent Y1 axis at E♭4, and a silent central minor sixth [8] span from B3 to G3:

(MP3 here)

66-STET (22-pitch) "all interval" scales derived from Carter's No.8 chord (Carter, 2002, p. 54).

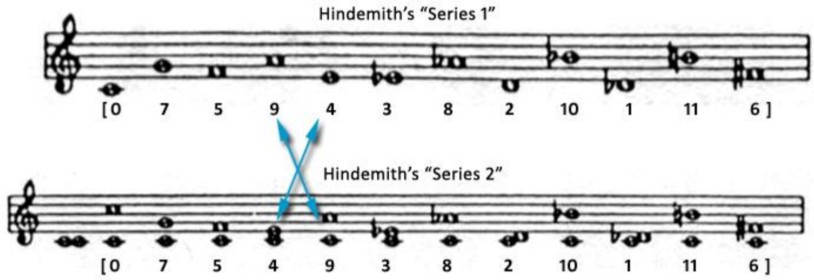


(MP3 here)

66-STET (22-pitch) adaptation of Hindemith's 1 and 2 series

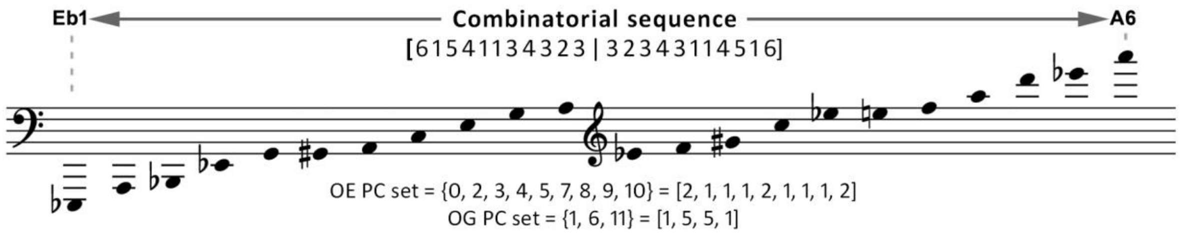
In *The Craft of Musical Composition* (1945), Hindemith's "Series 1" is derived from the overtone sequence, whereas "Series 2", Hindemith's "natural" order of intervals is derived from "combination tones [that] arise only when two or more tones sound simultaneously" (1945, Book I, p. 58). [94] Essentially, for Hindemith, the purpose of the two series is to stress the difference between harmonic and melodic "tension" within any (post-diatonic) chromatic system, where "any chord can occur in any key" (1945, Book I, p. 91).

[94] What Hindemith calls "first order" combination tones are the frequential difference between the two sounding pitches, in that "the simultaneous sounding of the tones c' (256 vibrations per second) and g' (384) produces the combination tone c (128), while the simultaneous sounding of c' (256) and f' (341.33) produces F (85.33)" (1945, Book I, p. 61). What he calls "second order" tones are calculated as the difference between the first order tone and the lower of the two notes of the interval. Hindemith uses A4 = 430Hz (approx.) rather than A4 = 435Hz as defined in ARTICLE 282 (22) in the Treaty of Versailles (1919), or today's "Stuttgart", "concert pitch" or midi standard A4 = 440Hz. Jos Kunst's online *Dissonant grading application* (Kunst, 2023) defines the Hindemith root for all chordal combinations.



Adapted from Fondation Hindemith, 2017

In Book II, Hindemith outlines how the chromatic tones of Series 1 are arranged in an "order of subordination" from a tonal centre (in this case 'C'); his order of subordination is "C: G, F, A, E, E♭, A♭, D, B♭, D♭, B, F♯" (1945, p. 91) – the intervallic set (from 'C') is thus [7, 5, **9, 4**, 3, 8, 2, 10, 1, 11, 6], whereas, the intervallic set of Series 2 is [7, 5, **4, 9**, 3, 8, 2, 10, 1, 11, 6]. The only difference between the two sets is the relative placement of the major third [4] and the sixth [9]; a descending minor third [3]. This can be related to the harmonic dualist's major/minor ↑↓ [3, 4] or [4, 3] tertian dichotomy. By interpreting Hindemith's scales in a centrosymmetric "LIE" format, and replacing [9] with [4, 5] (in both directions), this dichotomy can be integrated or ironed out. I.e. the ascending 66-STET intervallic sequence (here drawn around a silent C4 axis) becomes [7, 5, **4, 5, 4**, 3, 8, 2, 10, 1, 11, 6], with the descending variant being [6, 11, 1, 10, 2, 8, 3, **4, 5, 4**, 5, 7]. The combinatorial P+I sequence is as follows:



(MP3 here)

66-STET (33-pitch) scale derived from Carter's chords No.2 and 4; used in *Structural Declaration No.1 (*2021) (see Carter, 2002, p. 54).

(MP3 here)

66-STET (35-pitch) prime scale; used in *Vincent and the Maverick Sonority* (2021)

(MP3 here)

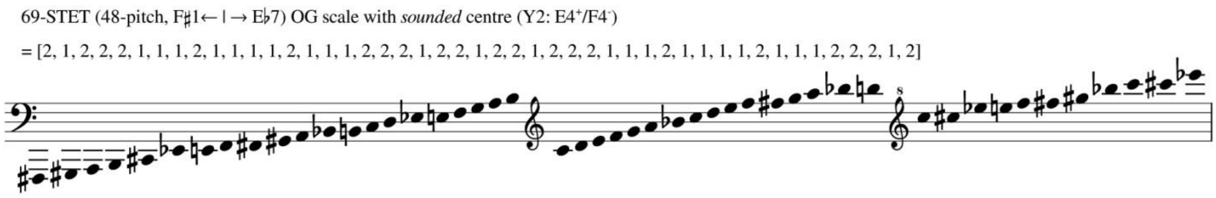
68-STET (13-pitch) scale; used in *3313133* (2021)

A subset of the 86-STET scale derived from [6, 5, 6 + 6, 5, 6] ↑↓ extensions from E♭4.

(MP3 here)

69-STET (48-pitch) scale

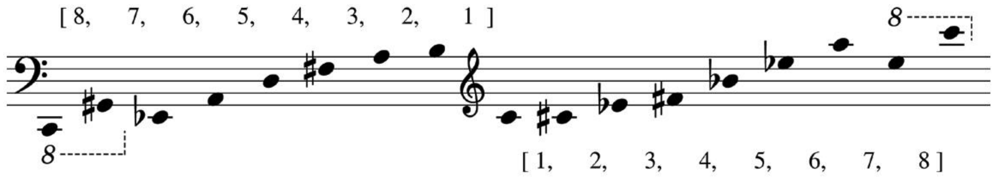
The OG scale used in *74 Pitches, 4 Hands, 1 Piano* (2017)



(MP3 here)

72-STET (17-pitch) scale

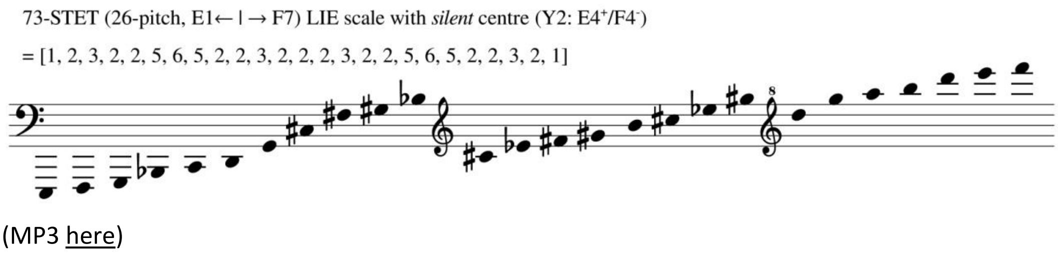
As used in *16 Hz the Threshold* (2020), this six octave scale extends outwards from a C4 (Y1) axial centre (C1 [36] C4 [36] C7).



(MP3 here)

73-STET (26-pitch) scale

The prime scale used in *74 Pitches, 4 Hands, 1 Piano* (2017)



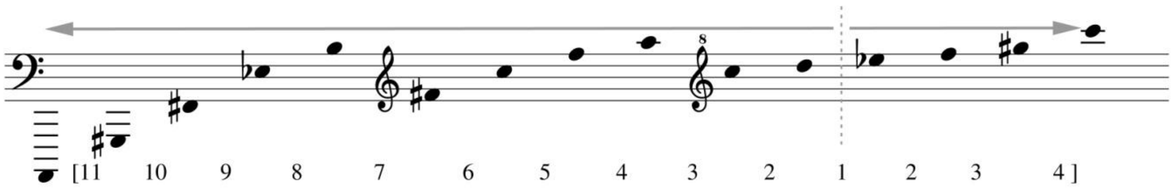
(MP3 here)

The ↑intervallic pattern [1, 2, 3, 2, 2, 5, 6, 5, 2, 2, 3, 2] is unique to the "Concatenation of subsequences..." (OEIS: A136436).

74-STET (B0 to C♯7) 44-pitch OG scale; used in *16 Hz The Threshold* (2020) and *Symphony No.1* (2020)

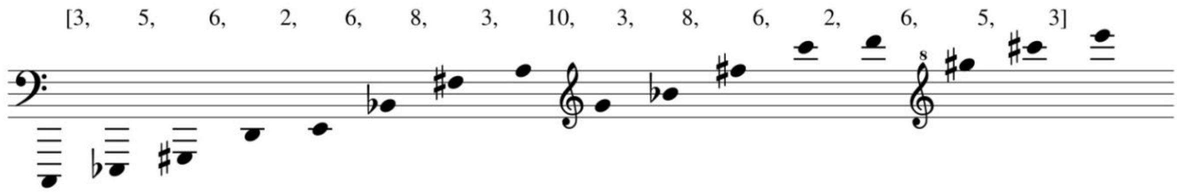
(MP3 here)

75-STET (15-pitch) scale; used in *3′ 44″ Duet for Xylophone and Piano* (2019)



(MP3 here)

76-STET (16-pitch) scalic extension of Gamer's 19-TET "Deep Scale" (1967a, p. 117)



(MP3 here)

76-STET Perrett scale

According to the figures given in Joel Mandelbaum's PhD thesis (1961), [95] the 19-TET systems proposed by both Ariel and Wilfred Perrett are centrosymmetric with regard to the distribution of micro intervallic distance, measured in cents (see diagram below).

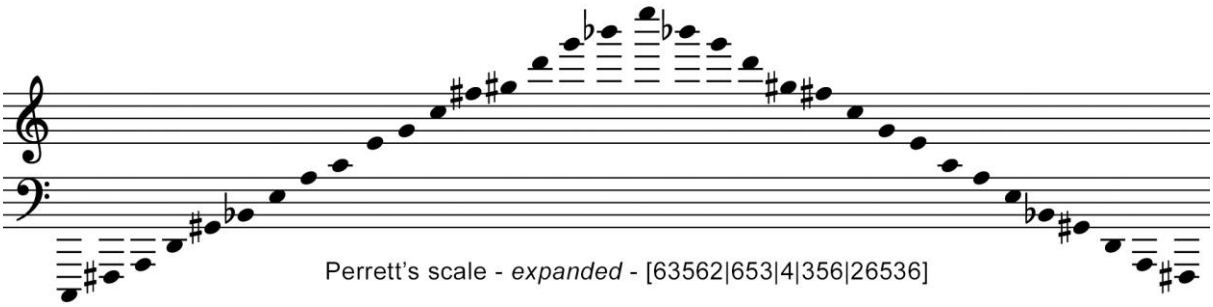
[95] Eight of Mandelbaum's 19 *cent* values for Wilfred Perritt's scale differ from those given by Partch (1974, p. 445).

	10-tones	Ariel	Brechmidt 1	2	Fokker	¼-comma	31-tone	Perrett	Plain	Septimal
D	0.0	0.0	.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
D♯	63.2	70.7	.7	70.7	70.7	76.0	77.4	85	70.7	71
E♭	126.3	111.7	1.7	133.2	111.7	117.1	116.1	134	133.2	119
E	189.5	182.3	3.9	203.9	182.3	193.2	193.5	204	182.3	204
E♯ or F♭	252.7	253.1	4.6	274.6	274.6	269.2	271.0	289	253.1	267
F	315.8	315.6	5.6	315.6	315.6	310.3	309.7	316	315.6	316
F♯	379.0	386.3	5.3	386.3	386.3	386.3	387.1	401	386.3	386
G♭	442.1	427.4	7.4	427.4	427.4	427.4	425.8	471	427.4	435
G	498.0	498.0	8.0	498.0	498.0	503.4	503.2	520	498.0	498
G♯	568.4	568.7	3.7	568.7	568.7	579.5	580.6	583	568.7	582
A♭	631.6	631.3	1.3	631.3	609.2	620.5	619.4	632	631.3	631
A	694.7	702.0	2.0	702.0	702.0	696.6	696.8	702	702.0	702
A♯	757.9	772.6	2.6	772.6	772.6	772.6	774.2	787	751.2	765
B♭	821.0	813.7	3.7	813.7	813.7	813.7	812.9	814	813.7	814
B	884.2	884.4	4.4	884.4	884.4	889.7	890.3	899	884.4	884
B♯ or C♭	947.3	946.9	5.4	925.4	925.4	930.8	929.0	969	955.0	969
C	1010.5	1017.7	6.1	996.1	996.1	1006.8	1006.5	1018	1017.7	1018
C♯	1073.7	1088.3	8.3	1066.8	1066.8	1082.9	1083.9	1103	1066.8	1088
D♭	1136.8	1129.3	0.3	1120.3	1129.3	1124.0	1122.6	1130	1129.3	1137

Ariel	Perrett
0.0	0.0
70.7	85
111.7	134
182.3	204
253.1	289
315.6	316
386.3	401
427.4	471
498.0	520
568.7	583
631.3	632
702.0	702
772.6	787
813.7	814
884.4	899
946.9	969
1017.7	1018
1088.3	1103
1129.3	1130

(Image adapted from Mandelbaum, 1961, p. 314)

In Ariel's system there are 3 distinct distances (give or take 200th of a cent) between successive tones (here labelled from A to C); these are (A) 70.7, (B) 41, and (C) 62.5. The centrosymmetric arrangement of these intervals is (A) BAAC|ABAA|C|AABA|CAAB. Perrett's system has five exact divisions, (A) 27, (B) 49, (C) 63, (D) 70, and (E) 85. In this case the pattern is EBDEA|EDB|C|BDE|AEDBE (AD). Perrett's five intervallic distances can be proportionally reduced to A=2, B=3, C=4, D=5, and E=6 (*approx*. when divided by 15). EBDEA|EDB|C|BDE|AEDBE thus becomes 63562|653|4|356|26536 which adds up to a span of 76. When rendered as a 76-STET semi-tonal span, from (e.g.) C1 to E7, the following scale is produced; OE PC set = [2, 2, 1, 1, 1, 1, 2, 2] – Pecot's palindromic "JIBian" scale (2019, p. 101).



(MP3 here)

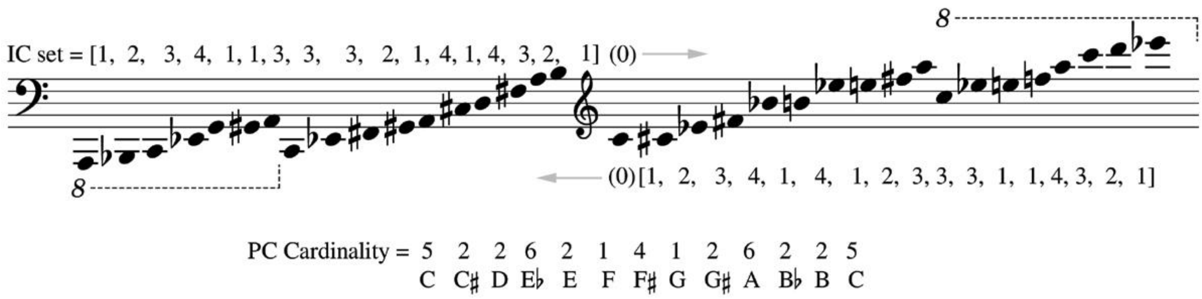
76-STET scale (C♯1 to F7) with a Y1 E♭4 axis; used in *Sketches for Player Piano, No.6* (2017)

(MP3 here)

78-STET (A0 to E♭7) 24-pitch scale; used in *16 Hz the Threshold* (2020)

(MP3 here)

78-STET (A0 to E♭7) 35-pitch scale; used in *16 Hz the Threshold* and *Symphony No.1* (2020)



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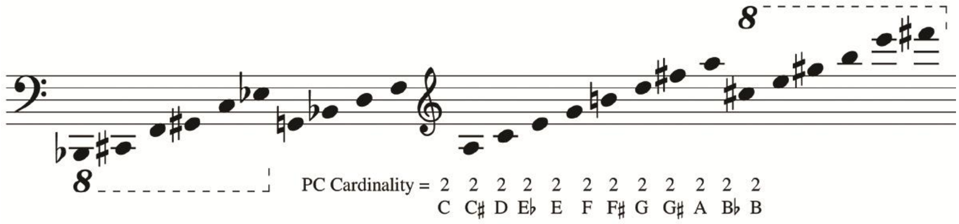
79-STET (50-pitch) C1 to G7: the OG scale used in *Symphony No. 9* (2024)

(MP3 here)

80-STET (B♭0 to F♯7) 24-pitch scale; an extension of the ↑iii + ↓iii mediants, from 'C4'



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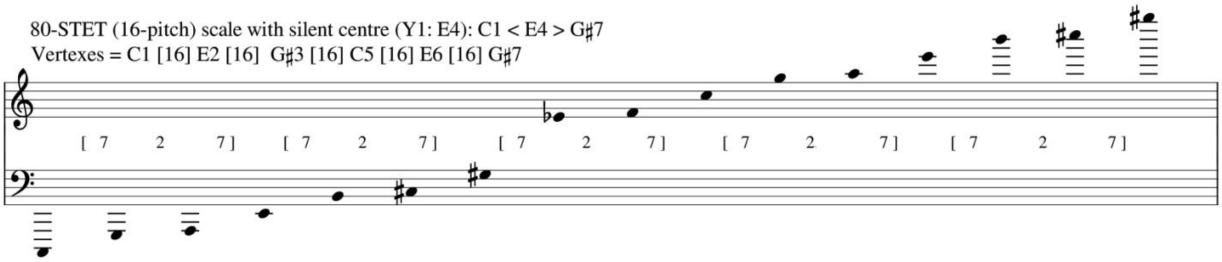
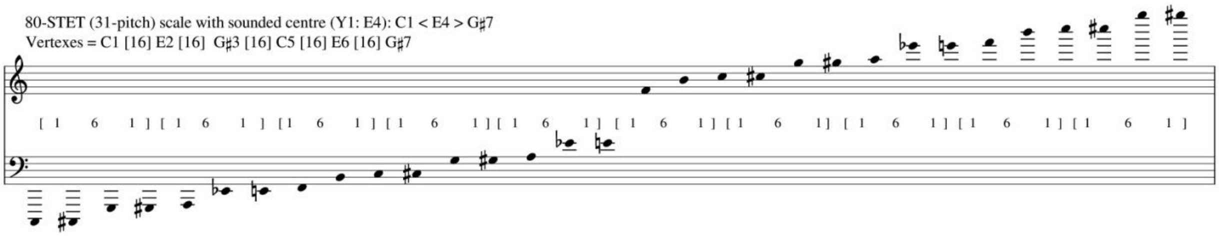
80-STET scalic examples

16-pitch extensions of a [7, 2, 7] 16-STET tetrachord.



(MP3 here)

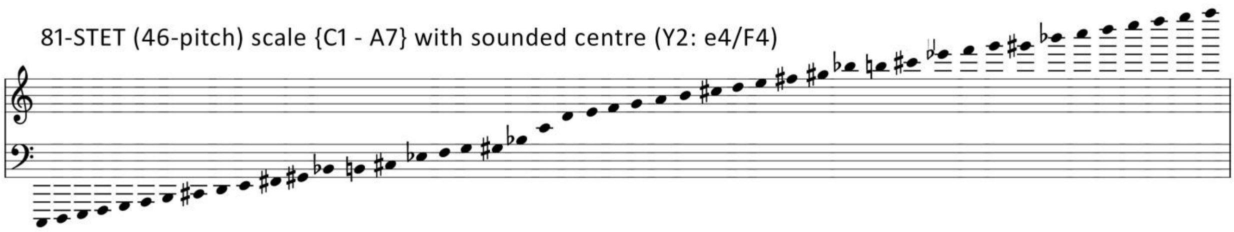
31-pitch extension of an 8-STET [1, 6, 1] tetrachord.



(MP3 here)

81-STET major sixth [2, 2, 1, 2, 2] extensions from C1 to A7

Extended variants of the central and centrosymmetric (major sixth) portion of the Lydian scale [2, 2, 1, 2, 2].

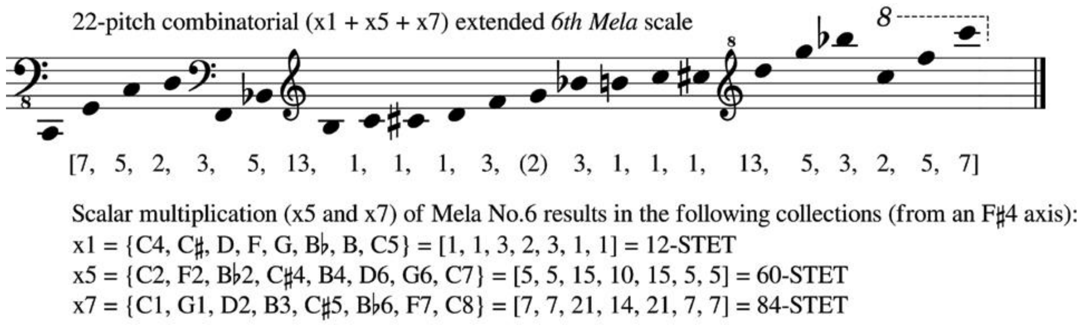


(MP3 here)

83-STET (64 pitches from B0 to B♭7); OG scale used in *13 piano études* (2018)

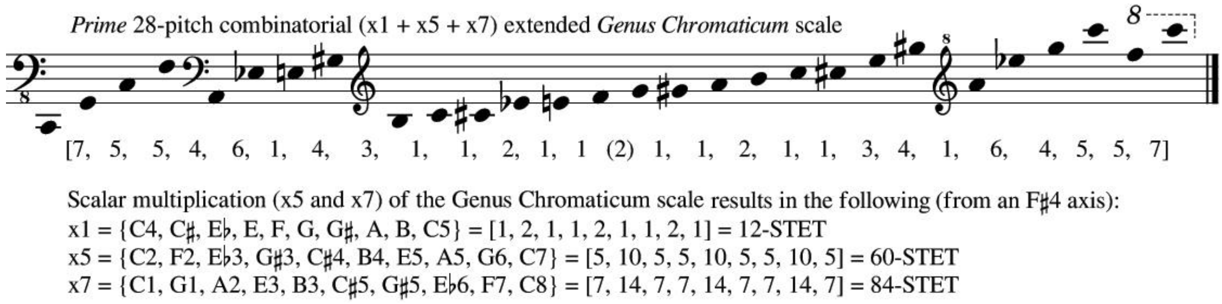
(MP3 here)

84-STET (22 pitches from C1 to C8); extended Mela no.6 scale used in *Symphony No.3* (2023)



(MP3 here)

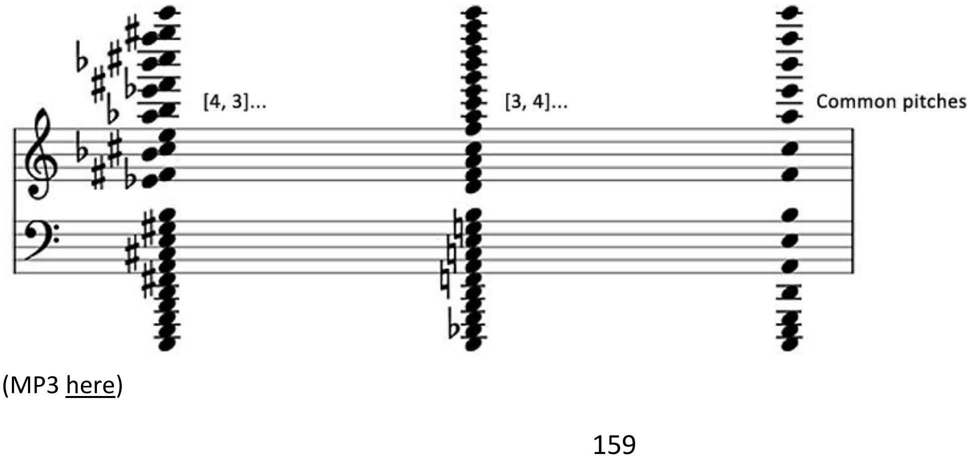
84-STET (28 pitches from C1 to C8); Prime scale used in *Symphony No.3* (2023)



(MP3 here)

84-STET symmetrical extensions of George Russell's *Lydian* scale

[4, 3] and [3, 4] extensions (stacks) of the [4, 3, 4, 3, 4, 3] {C, E, G, B, D, F♯, A} Lydian scale over an 84-STET (7 octave; C1 to C8) span.



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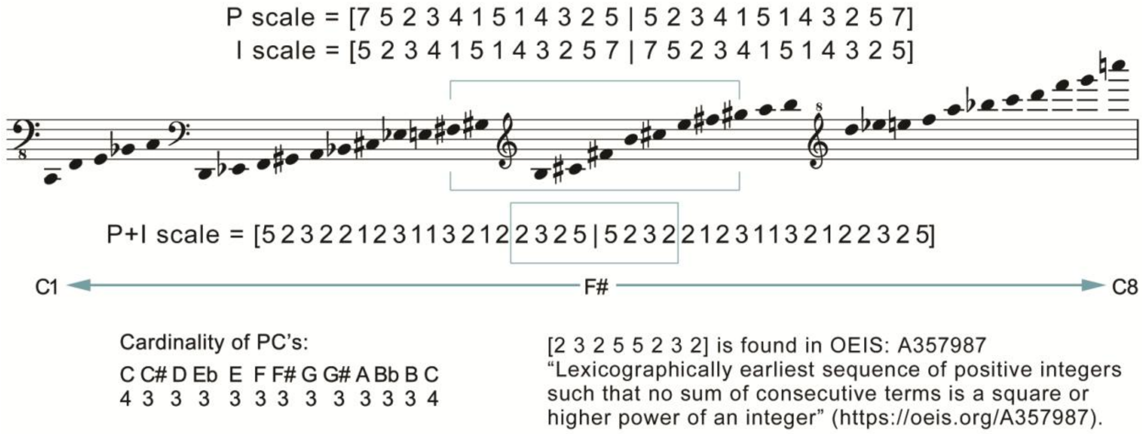
84-STET P+I "next of kin" construct from C1 to C8 with an F♯4 axis of symmetry



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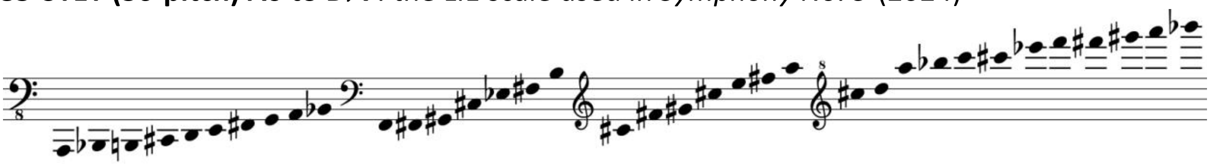
85-STET (36-pitch) A0 to B♭7: the LIE scale used in *Symphony No. 9* (2024)



(MP3 here)

85-STET (51-pitch) A0 to B♭7: the OG scale used in *3313133* (2021)

(MP3 here)

85-STET (64-pitch) A0 to B♭7; the OG scale used in *Outographic No.1* (2018)

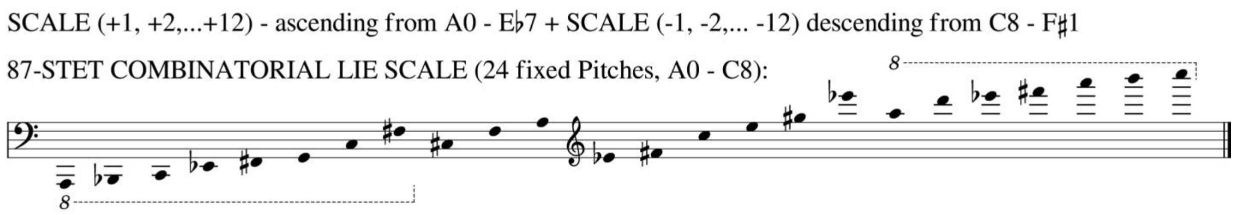
(MP3 here)

86-STET (37-pitch) scale; used in *3313133* (2021)

This scale has four silent Y2 axial points at C♯2/D2 - F♯3/G3 - B4/C5 - E6/F6.

(MP3 here)

87-STET (24-pitch) A0 to C8; used in *13 piano études* (2018)



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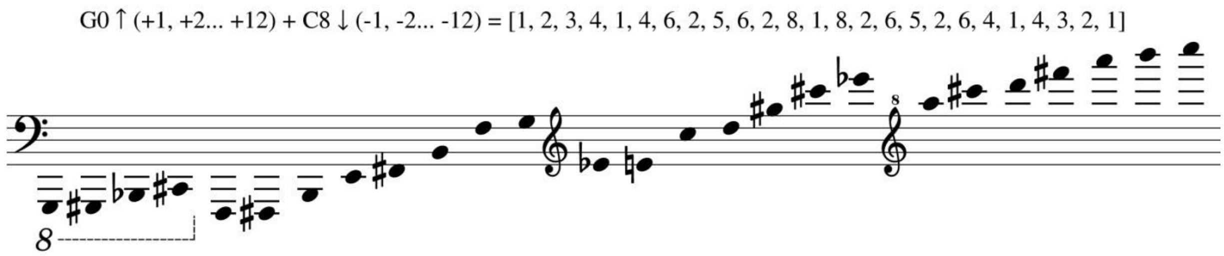
87-STET Complete chromatic from A0 to C8; used in *Queuing for gristle and bone* (2022).

(MP3 here)

89-STET (26-pitch) G0 to C8; the P+I generative LIE scale used in *Outographic No.1* (2018).



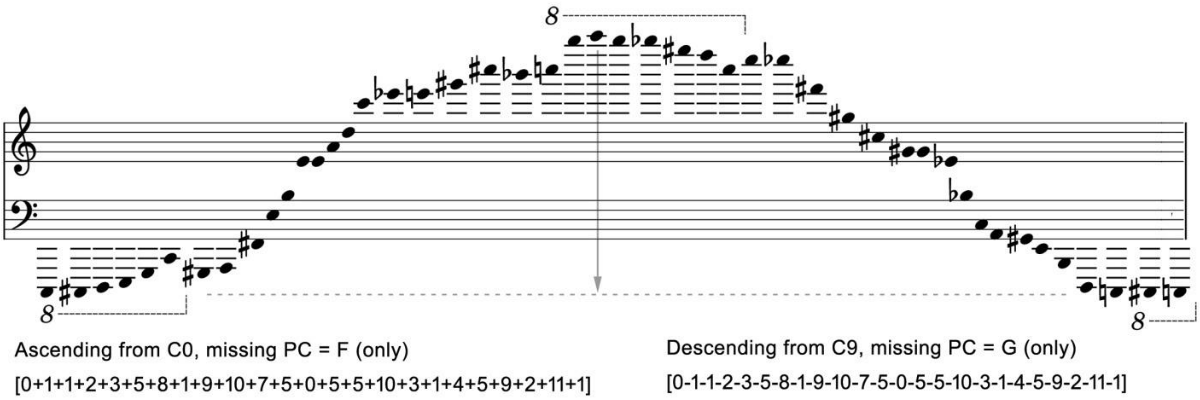
(MP3 here)

90-STET (25-pitch) scale, after Ives
The following scale is produced by applying what Schillinger calls an "optical projection through extension of the ordinate" (1943, p. 244) to Ives's 45-STET [2, 3, 5, 7, 11, 7, 5, 3, 2] scale, x 2.

(MP3 here)

108-STET (23-pitch) extension of the Virahanka/Fibonacci (mod 12) sequence; the registral unfolding of the "next of kin" *fifth duality.*

When written as a mod 12 (dodecaphonic) sequence, Reinhard Zumkeller notes that the complete system is a set of "10 periodic sequences [which] exhausts all possible ordered dyads; that is, every possible combination of two pitches is found in these sets"(OEIS: A089901). When rendered as an extended (108-STET) LIE scalic sequence (+1, +1, +2, +3, +5..., starting from C = 0), the following recursive 23 pitch, 22 interval (11 + 11) pattern emerges:

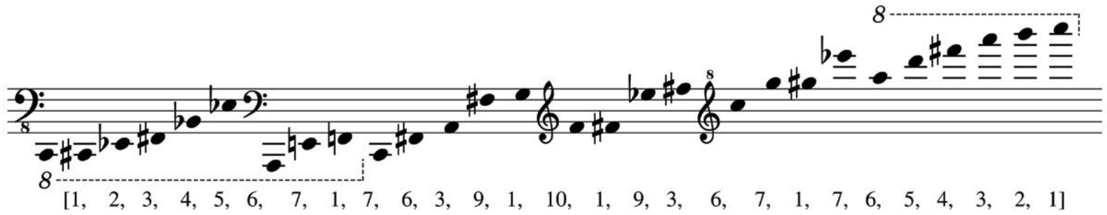


(MP3 here)

The only omitted PC from the above (P) ascending set is 'F', with 'G' being the only omitted PC within the descending (I) scale, an alternative embedded and extended representation of the "next of kin" fifth/fourth relationship. Perhaps, the further we stretch these generative pitch/integer components, the closer we get to cognitively appreciating some of the illusory or subliminal principles involved in auditory aesthetics, i.e. an underlying Leibnizian countability pertaining to Lewinian (intervallic) distance.

120-STET (28-pitch) C^{-1} to C^{9} scale; the source scale for *16 Hz the Threshold* (2020)

As the vertices extend beyond the instrumental registral range, this ten octave scale is a theoretical construct.



(MP3 here)

Appendix III – List of completed works composed using LIE scalic principles (2017-2024)

The scores, audio files and structural notes for all of these pieces can be found online at https://sonicimmersiontheory.com/appendix-iii/.

2017

Music for player piano (Nos. 1, 3, and 6)
Duration: 9’39”

28 Occasions of Experience
Scored for Clarinet and String Trio (Vln., Vla. and Vlc.)
Workshopped by the Distractfold Ensemble
Duration: 8’ 30”

74 pitches 4 hands 1 piano
Piano Duet
Duration: 4’ 24”

A gilded cage
Scored for Solo Flute
Performed by Amelie Donovan
Duration: 5’ 00”

As birds fly backwards
Scored for Flute and Electronics
Duration: 5’ 08” (including improvised section) 3’ 38” (without)

2018

Outographic No.1
Scored for Cl., Bsn., Trp., Trb., Vln., Db. and Perc. (Glk. and Xyl.)
Trombone performed by Steven Mai, Clarinet by Naomi Bayley
Duration: 4’ 20”

Heisenberg on Bourbon Street
Scored for C Trumpet, B♭ Trumpet, Trombone and Tuba
Duration: 4’ 30”

Even (odd) Scale
Scored for Piccolo Trumpet, Bass Clarinet and Piano
Duration: 3’ 19”

13 piano études
Duration: 20′ 20″

2019

3′ 44″
Duet for Xylophone and Piano
Duration: 3′ 44″

Notes of Protest
Duet for Cello and (virtual or live) B♭ Clarinet
Cello performed by Michael G. Ronstadt
Duration: 2′ 12″

2020

16Hz the Threshold
Sampled instrumentation includes; Flute, B♭ Clarinet, Baritone Sax, Piano, Timps, Drums, and Bass (with sampled spoken words)
Duration: 5′ 38″

Symphony No.1 (leisurely answers to long neglected letters)
Scored for Full Orchestra
Duration: 18′ 12″

2021

3313133 (Parts I, II and III)
Scored for Marimba, Vibraphone and Piano
Duration: 7′ 15″

[1, 4] cells
Scored for Piano, Bongos and Electric Guitar
Duration: 1′ 00″

Vincent and the Maverick Sonority
Scored for Piano
Duration: 3′ 56″

Structural Declaration No.1 (a piano concerto)
Scored for Large Ensemble and Piano
Duration: 9′ 00″

2022

Queuing for gristle and bone
Scored for B♭ Trumpet and Piano
Both parts performed by Daniel de Gruchy-Lambert
Duration: 4′ 04″

Fanfaronade
Scored for 2 Alto Saxophones and 2 B♭ Trumpets
B♭ Trumpet parts performed by Daniel de Gruchy-Lambert
Duration: 5′ 37″

An Anthem for Libertatia (parts I, II and III)
Scored for Large Ensemble (Small Orchestra) and Choir (SATB)
Soprano and Alto parts performed by Katherine Ellis,
Bass and Tenor parts performed by the composer
Duration: 21′ 21″

Symphony No.2
Scored for 1 Picc., 3 Fl., 2 Ob., Cl., B.Cl., Bsn., CBsn., 2 F.Hrn., Tpt., Tbn., B.Tbn., Tmp., Sn., Cym., Guit., Pno., Synth (pre-recorded), Vln., Vla., Vc., Cb
Duration: 21′ 15″

2023

Symphonies Nos. 3-8 are a hexalogy of 15′ (approx.) chamber symphonies.

Symphony No.3 (Tutti Frutti Happyland)
Scored for Picc., Fl., Ob., Cl., Bar. Sax, Bsn., Cbsn., F.Hrn., Tpt., Tbn., B.Tbn., Tmp., Sn., Brk. Dr., R.Cym., Cym., Vib., Pno., Vln., Vla., Vc., Cb
Duration: 15′ 17″

Symphony No.4 (Scalic and Melodic Objets Trouvés)
Scored for Picc., Fl., Ob., Cl., B.Cl., Bsn., F.Hrn., Tpt., Tbn., Timp., Sn., Cabasa, Pno., xyl., Vln., Vla., Vc., Cb
Duration: 14′ 58″

Symphony No.5 (Scream and Dance at the End of Days)
Scored for Fl., Ob., Cl., B.Cl., Bsn., F.Hrn., Tpt., Tbn., Tmp., Cymb., Tmp. Bl., Pno., Vln, 1, Vln. 2, Vla., Vc., Cb
Duration: 16′ 06″

Symphony No.6 (Cheek Filler and the Stepford Wives)
Scored for Picc., Fl., Ob., E. Hn., B♭ Cl., T. Sax., Bsn., F Hn., B♭ Tpt., Tbn., Tba., Timp., Xyl., Vib., Sn. Dr., Cym., Hrp., Vlns. 1, Vlns. 2, Vlas., Vcs., Cbs.
Duration: 15′ 05″

Symphony No. 7 (Gaza in the Sunshine - Old Trafford in the Rain)
Scored for Picc., Fl., Ob., E. Hn., B♭ Cl., T. Sax., Bsn., F Hn., B♭ Tpt., Tbn., Tba., Timp., Glock., XMrm., Con., Cym., Clv., Cabs., Vlns. 1, Vlns. 2, Vlas., Vcs., Cbs.
Duration: 15′ 07″

Symphony No. 8 (*In Memory of Scooby*)
Scored for Picc., Fl., Ob., E. Hn., B♭ Cl., T. Sax., Bsn., F Hn., B♭ Tpt., Tbn., Tba., Timp., Pno., Vlns. 1, Vlns. 2, Vlas., Vcs., Cbs.
Duration: 14′ 40″

An Apology To The Ocean (Ymddiheuriad I'r Cefnfor)
Scored for Tenor Saxophone and Trombone
Performed by Naomi Bayley and Steven Mai
Duration: 6' 16"

2024

Biting the Bullet between Gritted Teeth (2024)
Written for Harmonica, Bass and Drum Kit
Performed by Matt Baker, Richard Gibbons, and the composer
Duration: 6′ 20″

String Quartet No. 1 (Where's Noah?)
Movement No. 1 (other movements are work in progress)
Duration: 4' 50"

Symphony No. 9 (The Virahāṅka-Fibonacci Numbers)
Four Movements - Scored for Picc., Fl., Ob., E. Hn., B♭ Cl., Bsn., F Hn., B♭ Tpt., Tbn., Tba., Timp., Cym., Glock. (Mrm., Vib.), Pno., Hrp., Vlns. 1, Vlns. 2, Vlas., Vcs., Cbs.
Duration: 21' 44"

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